A.5.13

Exercise A.5.13 Simplify the rational expression to lowest terms:

$$\frac{y^2-25}{2y^2-8y-10}.$$

Solution

The numerator y^2-25 is of the form $x^2-a^2=(x-a)(x+a)$, so $y^2-25=(y-5)(y+5)$. To factor $2y^2-8y-10$, we look for integers a and b such that $ab=(2)(-10)=-20$ and $a+b=-8$; we can take $a=2$ and $b=-10$. So

$$\begin{aligned}2y^2-8y-10 &= 2y^2+2y-10y-10\\ &= 2y(y+1)-10(y+1)=(2y-10)(y+1)\\ &= 2(y-5)(y+1).\end{aligned}$$

Hence

$$\frac{y^2-25}{2y^2-8y-10}=\frac{(y-5)(y+5)}{2(y-5)(y+1)}$$

$$=\boxed{\frac{y+5}{2(y+2)} \text{ if } y\neq 5.} \quad \square$$