A.5.35

Exercise A.5.35 Perform the indicated operation and simplify the result. Leave your answer in factored form:

$$\frac{\dfrac{x-2}{x+2}+\dfrac{x-1}{x+1}}{\dfrac{x}{x+1}-\dfrac{2x-3}{x}}$$

Solution

The LCM of the denominators $(x+2)$, $(x+1)$, $(x+1)$, and x is $(x+2)(x+1)x$. We multiply the numerator and denominator by this (that is, we use "Method 2"):

$$\left(\frac{\dfrac{x-2}{x+2}+\dfrac{x-1}{x+1}}{\dfrac{x}{x+1}-\dfrac{2x-3}{x}}\right)\left(\frac{(x+2)(x+1)x}{(x+2)(x+1)x}\right)$$

$$=\frac{\dfrac{x-2}{x+2}(x+2)(x+1)x+\dfrac{x-1}{x+1}(x+2)(x+1)x}{\dfrac{x}{x+1}(x+2)(x+1)x-\dfrac{2x-3}{x}(x+2)(x+1)x}$$

$$=\frac{(x-2)(x+1)x+(x-1)(x+2)x}{x^2(x+2)-(2x-3)(x+2)(x+1)} \quad \text{if } x\neq -2,\ x\neq -1,\ x\neq 0$$

$$=\frac{x\big((x-2)(x+1)+(x-1)(x+2)\big)}{(x+2)\big(x^2-(2x-3)(x+1)\big)} \quad \text{if } x\neq -2,\ x\neq -1,\ x\neq 0$$

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$$= \frac{x\left((x^2 - x - 2) + (x^2 + x - 2)\right)}{(x+2)\left(x^2 - (2x^2 - x - 3)\right)} \quad \text{if } x = -2,\ x \neq -1,\ x \neq 0$$

$$= \frac{x(2x^2 - 4)}{(x+2)(-x^2 + x + 3)} \quad \text{if } x = -2,\ x \neq -1,\ x \neq 0$$

$$= \frac{-2x(x^2 - 2)}{(x+2)(x^2 - x - 3)} \quad \text{if } x \neq -2,\ x \neq -1,\ x \neq 0$$

$$= \boxed{\frac{-2x(x^2 - 2)}{(x+2)(x^2 - x - 3)} \quad \text{if } x \neq -1 \text{ and } x \neq 0.}$$

Notice that we do not have to write "$x \neq -2$" since this is implied by the term $(x+2)$ in the denominator. Also, $x^2 - 2$ and $x^2 - x - 3$ cannot be factored over the integers. □