

Exercise A.5.39 Reduce the expression to lowest terms $\frac{x \cdot 2x - (x^2+1) \cdot 1}{(x^2+1)^2}$.

This is an expression that occurs in calculus (in the setting of the Quotient Rule for Differentiation).

Solution

We simplify

$$\frac{(x)(2x) - (x^2+1)(1)}{(x^2+1)^2} = \frac{2x^2 - x^2 - 1}{(x^2+1)^2}$$

$$= \frac{x^2 - 1}{(x^2+1)^2} = \boxed{\frac{(x-1)(x+1)}{(x^2+1)^2}}$$

Notice that x^2+1 is prime by Theorem A.3.B and so cannot be factored. \square