PRECALCULUS 1 (ALGEBRA) - TEST 1

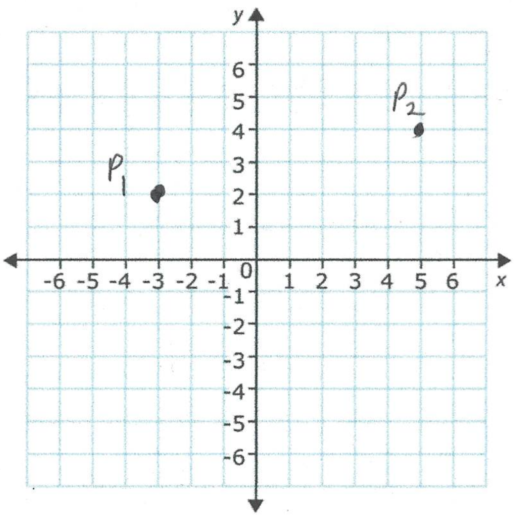
Fall 2019

NAME KEY STUDENT NUMBER 26

SHOW ALL WORK!!! Be clear and convince me that you understand what you are doing. Use equal signs when quantities are equal (and do not use them when things are not equal). Put your final answer in the box, when provided. Each numbered problem is worth 10 points. This test is two-sided and there are 10 questions. You can earn 1 bonus point per problem by clearly writing your answer using complete sentences! **No calculators, put away your cell phone and smart watch, and set the computer monitor face-down on the CPU!**

p.7 #25

1. Graph the points $P_1 = (-3, 2)$ and $P_2 = (5, 4)$. Find the distance between the two points.



With $P_1 = (x_1, y_1) = (-3, 2)$ and $P_2 = (x_2, y_2) = (5, 4)$ we have $x_1 = -3$, $y_1 = 2$, $x_2 = 5$, and $y_2 = 4$. So by the Distance Formula we have

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-(-3))^2 + (4-2)^2} = \sqrt{8^2 + 2^2}$$
$$= \sqrt{64+4} = \sqrt{68} = \sqrt{(4)(17)} = 2\sqrt{17}.$$

$$\boxed{\sqrt{68} = 2\sqrt{17}}$$

p. 7
#40

2. Find the midpoint of the line segment joining points $P_1 = (2, -3)$ and $P_2 = (4, 2)$.

With $P_1 = (x_1, y_1) = (2, -3)$ and $P_2 = (x_2, y_2) = (4, 2)$ we have $x_1 = 2$, $y_1 = -3$, $x_2 = 4$, and $y_2 = 2$. By the Midpoint Formula, the midpoint M is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{2+4}{2}, \frac{-3+2}{2} \right) = \left(\frac{6}{2}, \frac{-1}{2} \right) = \left(3, \frac{-1}{2}\right).$$

$$\boxed{\left(3, -\frac{1}{2}\right)}$$

3. Find the intercepts of the graph of the equation $9x^2 + 4y = 36$.

p.17
#29

For the x-intercepts, we set $y = 0$ and consider $9x^2 + 4(0) = 36$, or $9x^2 = 36$ or $x^2 = 36/9 = 4$. So $x = \pm 2$ and the x-intercepts are $x = -2$ and $x = 2$ (or the points $(-2, 0)$ and $(2, 0)$).
For the y-intercepts, we set $x = 0$ and consider $9(0)^2 + 4y = 36$, or $4y = 36$, or $y = 36/4 = 9$. So $y = 9$ and the y-intercept is $y = 9$ (or the point $(0, 9)$).

x-intercepts: $-2, 2$ y-intercepts: 9

p. 18
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4. Test the equation $y = \dfrac{x^4+1}{2x^5}$ for symmetry with respect to the y-axis, and the origin.

To test for symmetry with respect to the y-axis, we replace x with −x in the equation and see if we get an equivalent equation. Replacing x with −x gives the new equation $y = \dfrac{(-x)^4+1}{2(-x)^5} = -\dfrac{x^4+1}{2x^5}$. But with $x=1$, the original equation implies $y=1$ and in this new equation implies $y=-1$. So the equations are not equivalent and the equation is NOT symmetric WRT the y-axis.

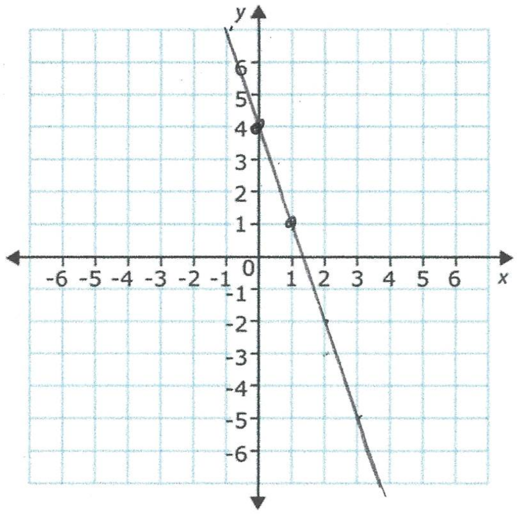
To test for symmetry with respect to the origin, we replace both x with −x and y with −y to get $(-y) = \dfrac{(-x)^4+1}{2(-x)^5}$ or $-y = -\dfrac{x^4+1}{2x^5}$ or $y = \dfrac{x^4+1}{2x^5}$.

Since this is the original equation, then the equation is symmetric WRT the origin.

$\boxed{\text{NOT symmetric WRT y-axis; IS symmetric WRT origin}}$

5. Find the slope and the y-intercept of the line $y = -3x + 4$ and graph the line.

p. 31
#74



Since $y=-3x+4$ is in slope-intercept form $y=mx+b$, then the slope is $m=-3$ and the y-intercept is $b=4$.

To graph the line, we find a second point on the line. Since the slope is $m = \dfrac{\Delta y}{\Delta x} = -3$, we can take $\Delta y = -3$ and $\Delta x = 1$ so that a second point on the line is (x,y) where $x = 0 + \Delta x = 0 + 1 = 1$ and $y = 4 + \Delta y = 4 + (-3) = 1$. Then a second point on the line is $(x,y) = (1,1)$. So the line is as graphed above.

$\boxed{\text{Slope } m=-3 \text{; y-intercept is } b=4}$

p. 31
#67

6. Find the equation of the line perpendicular to line $y = \frac{1}{2}x + 4$ such that the desired line contains the point $(-3, 0)$. Express your answer in slope-intercept form.

The given line is in Slope-Intercept Form then the given line has slope $m = \frac{1}{2}$. The desired line has slope $-1/m = -2$ and contains the point $(x_1, y_1) = (-3, 0)$, so from the Point-Slope Intercept Form we have that the desired line is $y - y_1 = m(x - x_1)$ or $y - 0 = -2(x - (-3))$ or $y = -2x - 6$. This is already in Point-Slope Form.

$y = -2x - 6$

7. Find the center and radius of the circle given by the equation $2(x - 3)^2 + 2y^2 = 8$.

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#25

The standard form for the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$ where the center is (h, k) and the radius is r, then $2(x-3)^2 + 2y^2 = 8$, or $(x-3)^2 + y^2 = 4$ has center $(h, k) = (3, 0)$ and radius $r = \sqrt{4} = 2$.

Center $(3, 0)$
Radius 2

p.57 #41

8. Determine whether the equation $2x^2 + 3y^2 = 1$ determines y as a function of x. Explain.

Solving for y we have $2x^2+3y^2=1$ or $3y^2 = 1-2x^2$, or $y^2 = \frac{1}{3}(1-2x^2)$, or $\sqrt{y^2} = \sqrt{\frac{1}{3}(1-2x^2)}$, or $|y| = \sqrt{\frac{1}{3}(1-2x^2)}$, or $y = \pm\sqrt{\frac{1}{3}(1-2x^2)}$.

With $x=0$ we have $y = \pm\sqrt{\frac{1}{3}(1-2(0)^2)} = \pm\sqrt{\frac{1}{3}}$, so with the input value $x=0$ we have multiple output values of $y = \sqrt{\frac{1}{3}}$ and $y = -\sqrt{\frac{1}{3}}$. So y is NOT a function of x.

$\boxed{y \text{ is NOT a function of } x.}$

9. Find the domain of the function $G(x) = \dfrac{x+4}{x^3-4x}$.

p.57 #58

We have $G(x) = \dfrac{x+4}{x^3-4x} = \dfrac{x+4}{x(x^2-4)} = \dfrac{x+4}{x(x-2)(x+2)}$.

Since we cannot divide by 0, then we must have $x(x-2)(x+2) \neq 0$ or $x \neq 0, x \neq 2, x \neq -2$. So the domain of G is all real numbers except $-2, 0, 2$. That is, the domain is $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$.

$\boxed{(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)}$

p. 65
#29 a, d

10. Consider the function $f(x) = \dfrac{2x^2}{x^4+1}$. **(a)** Is the point $(-1, 1)$ on the graph of f? (b) What is the domain of f?

(a) To see if the point $(-1, 1)$ is on the graph, we let $x = -1$ and see if $y = f(-1)$. We have $f(-1) = \dfrac{2(-1)^2}{(-1)^4+1} = \dfrac{2}{2} = 1$, so YES the point $(-1, 1)$ is on the graph of f.

(b) We cannot divide by 0 or take square roots of negatives. There are no square roots but there is division. But the denominator x^4+1 is always positive, so the domain of f is all real numbers $\mathbb{R} = (-\infty, \infty)$.

(a) YES
(b) $\mathbb{R} = (-\infty, \infty)$