PRECALCULUS 1 (ALGEBRA) - TEST 2

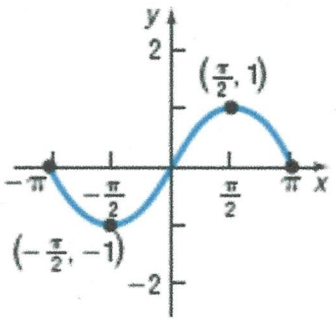
Fall 2019

NAME ___KEY___ STUDENT NUMBER ____________

SHOW ALL WORK!!! Be clear and convince me that you understand what you are doing. Use equal signs when quantities are equal (and do not use them when things are not equal). Put your final answer in the box, when provided. Each numbered problem is worth 11 points. This test is two-sided and there are 10 questions. You can earn 1 bonus point per problem by clearly writing your answer using complete sentences! **No calculators, put away your cell phone and smart watch, and set the computer monitor face-down on the CPU!**

1. Find the intercepts, if any, the domain, and range of the function with graph:

p79 #29ab



Since the points $(-\pi, 0)$, $(0,0)$, and $(\pi, 0)$ are the only points on the graph which are on the axes, then the x-intercepts are $-\pi, 0, \pi$ and the y-intercept is 0.
The domain is the set of x values where f is defined, so the domain is $[-\pi, \pi]$.
The range is the set of $y = f(x)$ values, so the range is $[-1, 1]$.

$\boxed{\text{x-intercepts } -\pi, 0, \pi \quad \text{y-intercepts } 0 \quad \text{Domain } [-\pi, \pi], \text{ Range } [-1, 1]}$

2. For the function in #1, find the intervals on which the function is increasing, decreasing, or constant. Determine whether the function is even, odd, or neither (and explain).

p79 #29cd

Since a function is increasing when it is going "uphill" (as read from left to right) then the function is increasing on $(-\frac{\pi}{2}, \frac{\pi}{2})$. A function is decreasing when it is going "downhill" so the function is decreasing on $(-\pi, -\frac{\pi}{2}) \cup (\pi/2, \pi)$
The function is symmetric with respect to the origin so it is an ODD function

$\boxed{\text{INC: } (-\pi/2, \pi/2) \quad \text{DEC: } (-\pi, -\pi/2) \cup (\pi/2, \pi) \quad \text{ODD}}$

p 90
#27

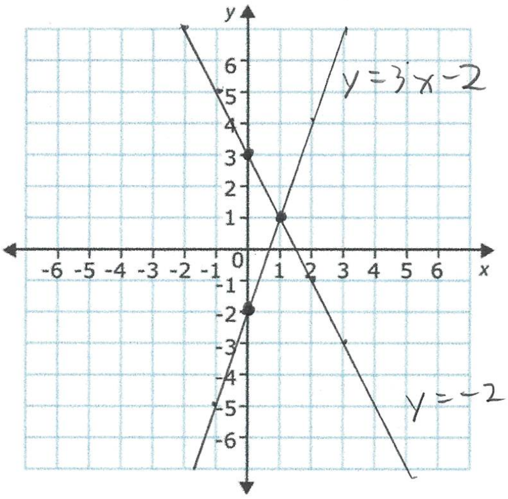
3. Consider the function $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ 2x+1 & \text{if } x > 0. \end{cases}$ Find $f(-2)$, $f(0)$, and $f(2)$.

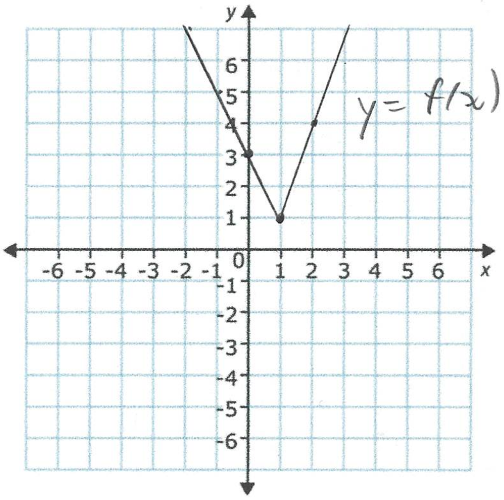
Since $-2 < 0$ we use the piece x^2 of f to get $f(-2) = (-2)^2 = 4$,
Since $0 = 0$ we use the piece 2 of f to get $f(0) = 2$,
Since $2 > 0$ we use the piece $2x+1$ of f to get $f(2) = 2(2)+1 = 5$

$$\boxed{f(-2) = 4,\ f(0) = 2 \quad f(2) = 5}$$

4. Consider the function $f(x) = \begin{cases} -2x+3 & \text{if } x < 1 \\ 3x-2 & \text{if } x \geq 1. \end{cases}$ Graph the function on the set of axes on the right and find the range of the function.

p 90
#33cd





We graph the lines $y = -2x+3$ and $y = 3x-2$ above. The y-intercept of $y = -2x+3$ is 3 and the point $(1, -2(1)+3) = (1, 1)$ is on the line so we use this information.
The y-intercept of $y = 3x-2$ is -2 and the point $(1, 3(1)-2) = (1, 1)$ is on the line so we use this information to graph.

On the right, we keep the appropriate piece of the two lines to produce the graph of f.

From the graph, we have

$$\boxed{\begin{array}{l} \text{domain } \mathbb{R} = (-\infty, \infty) \\ \text{range } [1, \infty) \end{array}}$$

p103
#27

5. Find the equation for the function $f(x)$ that results from ~~by~~ applying the following transformations (in order) to the graph of $y = \sqrt{x}$: (1) Shift up 2 units, (2) reflect about the x-axis, and (3) reflect about the y-axis. You do not need to graph the function.

To shift up 2 units we add +2 to $\sqrt{x}$ to get $y = \sqrt{x} + 2$. To reflect about the x axis we multiply $(\sqrt{x}+2)$ by -1 to get $y = -(\sqrt{x}+2)$. To reflect about the y-axis we replace x with $-x$ in $y = -(\sqrt{x}+2)$ to get $y = f(x) = -(\sqrt{-x}+2) = -\sqrt{-x} - 2$

$$\boxed{f(x) = -\sqrt{-x} - 2}$$

6. Graph the function $f(x) = (x-1)^3 + 2$ on the set of axes on the right by starting with the Library of Functions element $y = x^3$ and using the transformations of shifting, compression/stretching, and/or reflecting.

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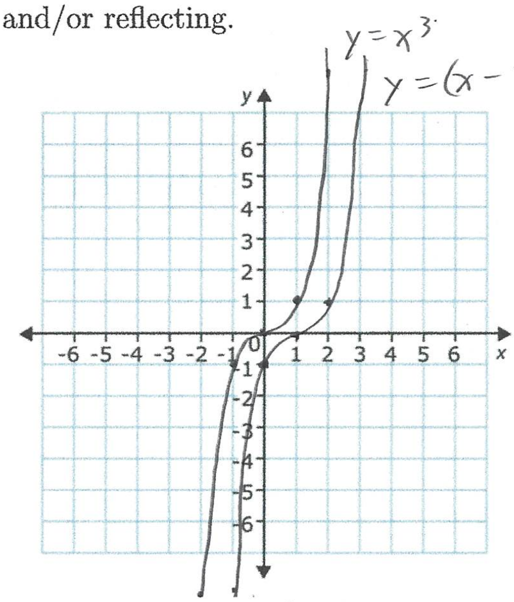
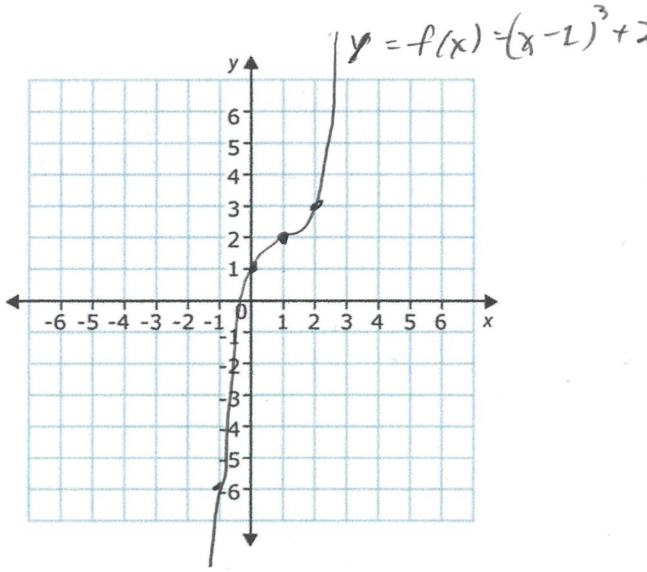
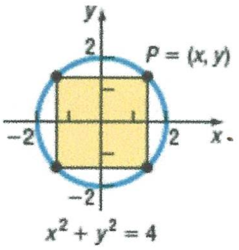


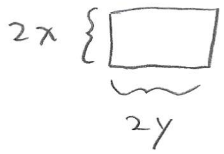


We start with $y = x^3$ and first replace x with $x-1$, which is a horizontal shift to the right by 1 unit, of $y = x^3$, to get $y = (x-1)^3$. Next we add 2 to $(x-1)^3$, which is a vertical shift up by 2 units of $y = (x-1)^3$, to get $y = (x-1)^3 + 2 = f(x)$.

p109
#9a

7. A rectangle is inscribed in a circle of radius 2. See the figure. Let $P = (x, y)$ be the point in quadrant I that is a vertex of the rectangle and is on the circle. Express the area of the rectangle as a function of x.





Notice that (x, y) is in Quadrant I so that $x > 0$ and $y > 0$. The width of the rectangle is $2y$ and the height is $2x$. So the area is $A = (\text{width})(\text{height}) = (2y)(2x)$.
Since $x^2 + y^2 = 4$ then $y^2 = 4 - x^2$ and $y = \pm\sqrt{4 - x^2}$. Since $y > 0$ then $y = \sqrt{4 - x^2}$.
So as a function of x, $A = 4xy = 4x\sqrt{4 - x^2}$

$\boxed{A(x) = 4x\sqrt{4 - x^2}}$

8. Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 5$. Solve (a) $f(x) = g(x)$, (b) solve $f(x) \geq g(x)$, and (c) graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution of the equation $f(x) = g(x)$.

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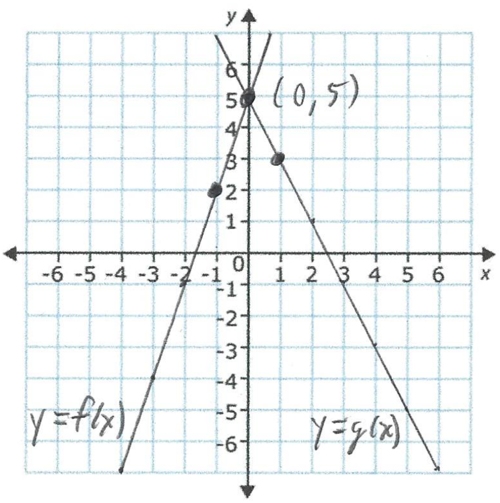


(a) $f(x) = g(x)$ implies $3x + 5 = -2x + 5$ or $5x = 0$ or $x = 0$.

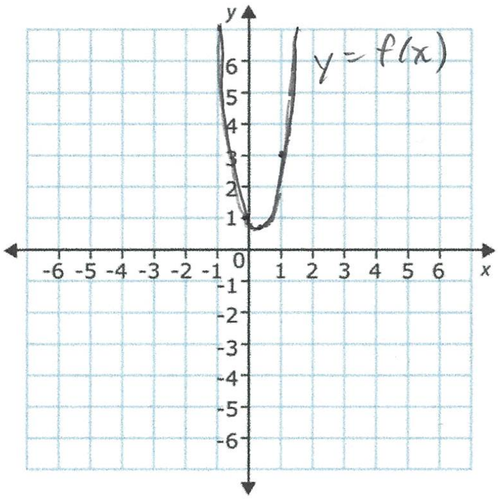
(b) $f(x) \geq g(x)$ implies $3x + 5 \geq -2x + 5$ or $5x \geq 0$ or $x \geq 0$.

(c) The intercept of the graph of each is 5. Another point on $f(x) = 3x + 5$ is $(-1, f(-1)) = (-1, 3(-1) + 5) = (-1, 2)$. Another point on $f(x) = -2x + 5$ is $(1, f(1)) = (1, -2(1) + 5) = (1, 3)$. Since these are linear functions, this is enough information to graph.

(a) $x = 0$
(b) $x \geq 0$

p146
#42a c

9. Consider $f(x) = 4x^2 - 2x + 1$. Graph $y = f(x)$ by determining whether its graph opens up or down, find its vertex and axis of symmetry. From the graph, determine where the function is increasing and where it is decreasing.



We have $a=4$, $b=-2$, $c=1$. Since $a=4>0$, the graph opens up. The vertex is $(-b/2a, f(-b/2a))$

$= (-(-2)/(2(4)), f(-(-2)/(2(4))))$

$= ((\frac{1}{4}), f(\frac{1}{4})) = (\frac{1}{4}, 4(\frac{1}{4})^2 - 2(\frac{1}{4}) + 1)$

$= (\frac{1}{4}, \frac{1}{4} - \frac{2}{4} + \frac{4}{4}) = (\frac{1}{4}, \frac{3}{4})$.

The axis of symmetry is

$x = \frac{-b}{2a} = \frac{1}{4}$.

From the graph, f is increasing on $(\frac{1}{4}, \infty)$ and decreasing on $(-\infty, \frac{1}{4})$.

$\boxed{\text{opens up, vertex } (\frac{1}{4}, \frac{3}{4}), \text{ axis } x = 1/4, \text{ INC } (\frac{1}{4}, \infty), \text{ DEC } (-\infty, \frac{1}{4}).}$