PRECALCULUS 1 (ALGEBRA) - TEST 3

Fall 2019

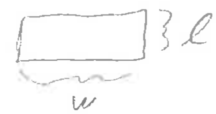
NAME KEY STUDENT NUMBER ______________

SHOW ALL WORK!!! Be clear and convince me that you understand what you are doing. Use equal signs when quantities are equal (and do not use them when things are not equal). Put your final answer in the box, when provided. Each numbered problem is worth 12 points. This test is two-sided and there are 8 questions. You can earn 1 bonus point per problem by clearly writing your answer using complete sentences! **No calculators, put away your cell phone and smart watch, and set the computer monitor face-down on the CPU!**

1. David has 400 yards of fencing and wishes to enclose a rectangular area. Express the area A of the rectangle as a function of the width w of the rectangle. For what value of w is the area largest (explain)?

$\boxed{\text{p 154 \#7 a, b}}$

Let l be the length and w the width of the field:



Then the perimeter (amount of fencing) is $2w+2l=400$ yards, so $w+l=200$ and $l=200-w$. The area is $A=wl=w(200-w)$ and hence $A(w)=200w-w^2$. This is a quadratic function with $a=-1$, $b=200$, and $c=0$ so the graph of A opens down (since $a=-1<0$) and A has a MAX at $w=-b/(2a)=-(200)/(-2)=100$ yards.

$\boxed{\begin{array}{l} A(w)=200w-w^2 \\ \text{MAX at } w=100 \text{ yards} \end{array}}$

2. Solve the inequality $6x^2 < 6+5x$ and express your answer in interval notation. HINT: $6x^2-5x-6=(2x-3)(3x+2)$ and the quadratic equation states $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$\boxed{\text{p 160 \#16}}$

Inequality $6x^2<6+5x$ is equivalent to $6x^2-5x-6<0$. With $f(x)=6x^2-5x-6$ we consider $f(x)<0$. Now $f(x)=(2x-3)(3x+2)$ so the roots of f are $x=3/2$ and $x=-2/3$. Consider:

	$\left(-\infty, \frac{-2}{3}\right)$	$\left(\frac{-2}{3}, \frac{3}{2}\right)$	$\left(\frac{3}{2}, \infty\right)$
c	-1	0	2
$f(c)$	$(-5)(-2)$	$(-3)(2)$	$(1)(8)$
$f(x)$	$+$	$-$	$+$

So $f(x)<0$ for $x\in\left(\frac{-2}{3}, \frac{3}{2}\right)$

$\boxed{\left(\frac{-2}{3}, \frac{3}{2}\right)}$

3. Consider the function $f(x) = x^2(x-2)(x+2)$. Find the end behavior and the intercepts (if any) of the function.

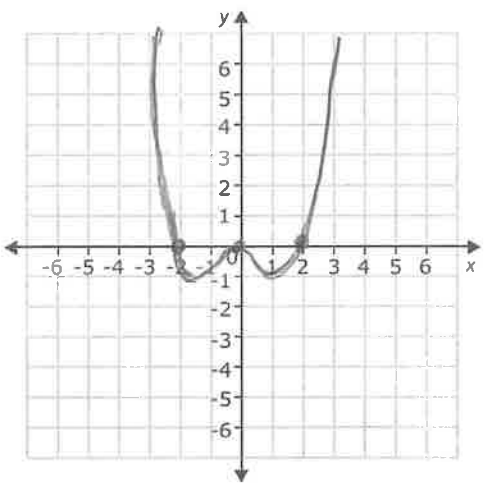
p186
#89a,b

We have $f(x) = x^2(x^2-4) = x^4 - 4x^2$, so the end behavior is $y = x^4$. The x-intercepts are obtained for $f(x) = x^2(x-2)(x+2) = 0$ and so are $x=-2$, $x=0$, $x=2$. For the y-intercept we set $x=0$ to get $y = f(0) = 0$.

End Behavior $y = x^4$
x-int : $-2, 0, 2$
y-int : 0

4. Consider the function $f(x) = x^2(x-2)(x+2)$ again. Find the roots of f, determine whether it crosses or touches the x-axis at these roots, and graph $y = f(x)$.

p186
#89c,d



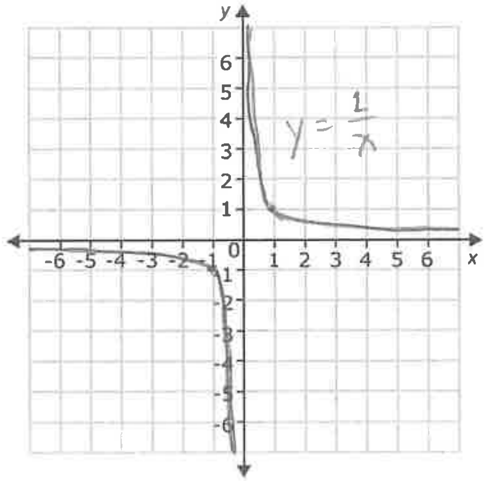
The roots of f are solutions to $f(x)=0$ and so are $x = -2, 0, 2$ (as above).
Consider

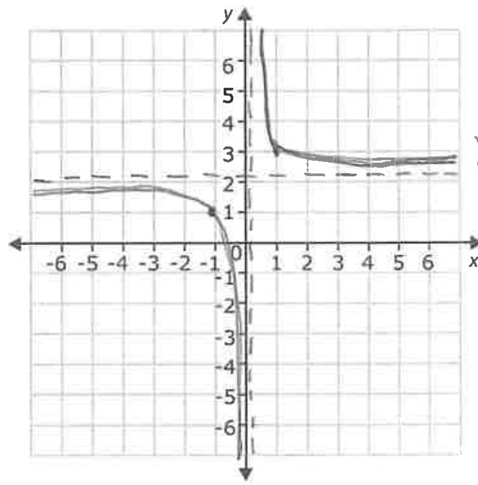
	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
c	-3	-1	1	3
$f(c)$	$9(-5)(-1)$	$(1)(-3)(1)$	$(1)(-1)(3)$	$(9)(1)(5)$
$f(x)$	+ above x-axis	− below x-axis	− below x-axis	+ above x-axis

Roots : $-2, 0, 2$
Crosses at $x = -2, 2$
Touches at $x = 0$

5. Consider $F(x) = 2 + \dfrac{1}{x}$. Start with the Library of Functions graph $y = 1/x$ and use transformations to graph $y = F(x)$. What is the domain, range, and any asymptotes of F?

p 197 #33





We start with $y = 1/x$ and add 2 to translate the graph of $y = 1/x$ up 2 units to get $y = \frac{1}{x} + 2$.

The domain excludes $x = 0$, the range excludes $y = 2$. Horizontal asymptote $y = 2$. Vertical asymptote $x = 0$

$$\boxed{\begin{array}{l} \text{Domain } (-\infty, 0) \cup (0, \infty) \\ \text{Range } (-\infty, 2) \cup (2, \infty) \\ \text{H.A. } y = 2, \ \text{V.A. } x = 0 \end{array}}$$

6. Consider $G(x) = \dfrac{x}{(x+2)^2}$. Find the domain of G, the intercepts, vertical asymptotes, and horizontal asymptotes (if they exist).

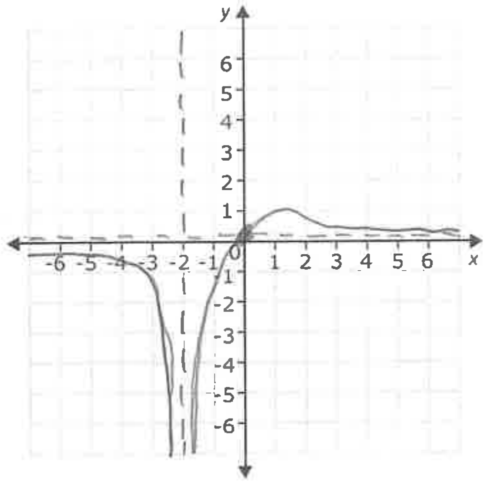
p 211 #43

This is a rational function; the domain is all real numbers except where the denominator is 0, so domain is $(-\infty, -2) \cup (-2, \infty)$. The x-intercept is found from $G(x) = 0$ and so is $x = 0$. The y-intercept is $y = G(0) = 0$. Since G is in lowest terms, the V.A. is $x = -2$. Since the numerator is degree 1 and the denominator is degree 2 then $y = 0$ is the H.A.

$$\boxed{\begin{array}{l} \text{Domain } (-\infty, -2) \cup (-2, \infty) \\ x\text{-int}: x = 0 \\ y\text{-int}: y = 0 \\ \text{V.A. } x = -2, \ \text{H.A. } y = 0 \end{array}}$$

p211
#43

7. Consider $G(x) = \dfrac{x}{(x+2)^2}$ again. Where is the graph of $y = G(x)$ above and where is it below the x-axis? Graph $y = G(x)$.



We eliminate the roots of the numerator and denominator from $(-\infty, \infty)$ and consider

	$(-\infty, -2)$	$(-2, 0)$	$(0, \infty)$
c	-3	-1	1
$G(c)$	$(-3)/(-1)^2$	$(-1)/(1)^2$	$(1)/(3)^2$
$G(x)$	$-$ below x-axis	$-$ below x-axis	$+$ above x-axis

Below x-axis:
$(-\infty, -2) \cup (-2, 0)$
Above x-axis:
$(0, \infty)$

p219
#35

8. Solve the inequality $\dfrac{(x-1)(x+1)}{x} \le 0$ and express your answer in interval notation.

The roots of the numerator are -1 and 1, and the root of the denominator is 0. So define $f(x) = (x-1)(x+1)/x$ and consider

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
c	-2	$-1/2$	$1/2$	2
$f(c)$	$(-3)(-1)/(-2)$	$(\frac{-3}{2})(\frac{1}{2})/(\frac{-1}{2})$	$(\frac{-1}{2})(\frac{3}{2})/(\frac{1}{2})$	$(1)(3)/(2)$
$f(x)$	$-$ below x-axis	$+$ above x-axis	$-$ below x-axis	$+$ above x-axis

$(-\infty, -1] \cup (0, 1]$

So $f(x)$ is negative on $(-\infty, -1) \cup (0, 1)$ and equal to 0 at $x = -1$ and $x = 1$.
Hence $f(x) \le 0$ for $x \in (-\infty, -1] \cup (0, 1]$.