PRECALCULUS 1 (ALGEBRA) - TEST 4
Fall 2019

NAME KEY STUDENT NUMBER ______________

SHOW ALL WORK!!! Be clear and convince me that you understand what you are doing. Use equal signs when quantities are equal (and do not use them when things are not equal). Put your final answer in the box, when provided. Each numbered problem is worth 12 points. This test is two-sided and there are 8 questions. You can earn 1 bonus point per problem by clearly writing your answer using complete sentences! **No calculators, put away your cell phone and smart watch, and set the computer monitor face-down on the CPU!**

$\boxed{p233 \ \#53}$

1, 2. Consider $f(x) = x^4 + x^3 - 3x^2 - x + 2$. Use the Rational Zeros Theorem to find all possible rational zeros of f. How many real zeros may f have? Find all real zeros of f and factor f.

We have a 4th degree polynomial function, so f has at most 4 zeros. Here, $a_0 = 2$ and $a_n = a_4 = 1$, so by the Rational Zeros Theorem, the possible zeros of f are p/q where p is a divisor of $a_0 = 2$ and q is a divisor of $a_4 = 1$. So the possible rational zeros are $\pm 1, \pm 2$. We have $f(1) = 0$, $f(-1) = 0$, $f(2) = 12$, $f(-2) = 0$, and so $1, -1, -2$ are zeros and $(x-1), (x+1), (x+2)$ are factors. Notice $(x-1)(x+1)(x+2) = (x^2-1)(x+2) = x^3+2x^2-x-2$ is a factor of f so we perform long division:

$$
\begin{array}{r}
x - 1 \\
x^3+2x^2-x-2 \overline{\big) x^4 + x^3 - 3x^2 - x + 2} \\
-(x^4 + 2x^3 - x^2 - 2x) \\
\hline
-x^3 - 2x^2 + x + 2 \\
-(-x^3 - 2x^2 - x - 2) \\
\hline
0
\end{array}
$$

So $x-1$ is also a factor of f, and $f(x) = (x-1)^2(x+1)(x+2)$.

$\boxed{\begin{array}{l}\text{Possible rational zeros: } \pm 1, \ \pm 2 \\ f \text{ has at most 4 real zeros} \\ \text{Real zeros: } 1 \text{ (of multiplicity 2)}\end{array}}$

-1, and -2

$f(x) = (x-1)^2(x+1)(x+2)$

p255 #36

3. Consider $f(x) = x^2 + 4$ and $g(x) = \sqrt{x-2}$. Find the composite functions $f \circ g$ and $g \circ f$. State the domain of each.

We have $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-2}) = (\sqrt{x-2})^2 + 4$ for $x \geq 2$

$= (x-2) + 4 = x + 2$ for $x \geq 2$, and

$(g \circ f)(x) = g(f(x)) = g(x^2+4) = \sqrt{(x^2+4) - 2} = \sqrt{x^2 - 2}$ for $x^2 \geq 2$

$= \sqrt{x^2-2}$ for $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$\boxed{(f \circ g)(x) = x+2 \text{ for } x \geq 2 \quad (g \circ f)(x) = \sqrt{x^2-2} \text{ for } x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)}$

p267 #58ab

4. Consider the one-to-one function $f(x) = x^2 + 9$ where $x \geq 0$. Find its inverse f^{-1}. Find the domain and the range of f and f^{-1}.

To find $f^{-1}(x)$, set $y = f(x)$, interchange x and y, and solve for y:

$y = x^2 + 9,\ x \geq 0 \Rightarrow x = y^2 + 9,\ y \geq 0$ or $y^2 = x - 9,\ y \geq 0$

or $\sqrt{y^2} = \sqrt{x-9},\ y \geq 0$ or $|y| = \sqrt{x-9},\ y \geq 0$ or $y = \sqrt{x-9}$, so

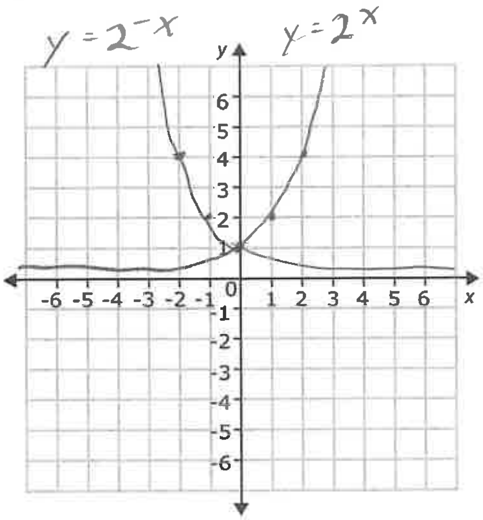
$y = f^{-1}(x) = \sqrt{x-9}$. The domain of f is $x \geq 0$, so the range of f^{-1} is $[0, \infty)$. The domain of f^{-1} is $x \geq 9$, so the range of f is $[9, \infty)$.

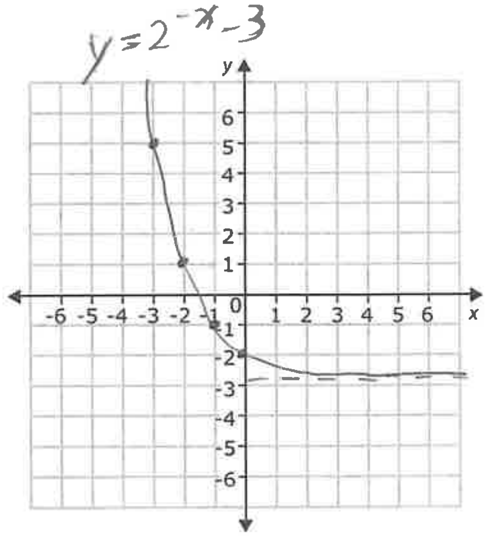
$\boxed{f^{-1}(x) = \sqrt{x-9} \quad \text{domain of } f: [0, \infty) \quad \text{range of } f: [9, \infty)}$

domain of f^{-1}: $[9, \infty)$
range of f: $[0, \infty)$.

p276
Ex. 5

5, 6. Consider $f(x) = 2^{-x} - 3$. Start with the Library of Functions graph of $y = 2^x$ and use transformations to graph $y = f(x)$ (and explain). Determine the domain, range, and horizontal asymptote of f.





We start with $y = 2^x$ and replace x with $-x$; this gives a transformation of $y = 2^x$ of a reflection about the y-axis. Next, we add -3 to $y = 2^{-x}$ to get $f(x) = 2^{-x} - 3$; this gives a transformation of $y = 2^{-x}$ of a shift down by 3 units.

From the graph, we see that the domain of f is $(-\infty, \infty)$, the range is $(-3, \infty)$, and $y = -3$ is the horizontal asymptote.

domain	$(-\infty, \infty)$
range	$(-3, \infty)$
H.A.	$y = -3$

p295
#34,36

7. Find the exact value of: **(a)** $\log_5 \sqrt[3]{25}$ and **(b)** $\log_{\sqrt{3}} 9$.

(a) We have $\sqrt[3]{25} = 5^{2/3}$, so $y = \log_5(\sqrt[3]{25}) = \log_5(5^{2/3})$ is equivalent to the exponential equation $5^y = 5^{2/3}$ so that $y = \log_5(\sqrt[3]{25}) = 2/3$,

(b) We have $9 = 3^2 = (\sqrt{3}^2)^2 = \sqrt{3}^4$, so $y = \log_{\sqrt{3}}(9) = \log_{\sqrt{3}}(\sqrt{3}^4)$ is equivalent to the exponential equation $\sqrt{3}^y = (\sqrt{3})^4$ so that $y = 4$.

(a)	$2/3$
(b)	4

p305
#56

8. Write $\ln\left(\dfrac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right)$, where $0 < x < 1$, as a sum and/or difference of logarithms. Express powers as factors.

By the properties of logarithms we have

$$\ln\left(\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right) = \ln(5x^2\sqrt[3]{1-x}) - \ln(4(x+1)^2)$$

$$= \ln 5 + \ln x^2 + \ln(1-x)^{1/3} - \ln 4 - \ln(x+1)^2$$

$$= \ln 5 + 2\ln x + \tfrac{1}{3}\ln(1-x) - \ln 4 - 2\ln(x+1)$$

$$\boxed{\ln(5) + 2\ln x + \tfrac{1}{3}\ln(1-x) - \ln 4 - 2\ln(x+1)}$$