PRECALCULUS 1 (ALGEBRA) - TEST 1

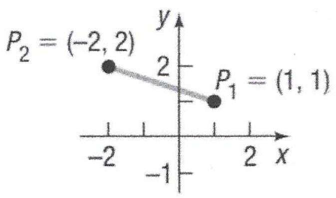
Fall 2021

NAME KEY STUDENT NUMBER ____________________

SHOW ALL WORK!!! Be clear and convince me that you understand what you are doing. Use equal signs when quantities are equal (and do not use them when things are not equal). **Write in complete sentences** and explain your answer. Put your final answer in the box, when provided. Each numbered problem is worth 11 points. This test is two-sided and there are 9 questions. **No calculators, put away your cell phone and smart watch, and set the computer monitor face-down on the CPU!**

1. Find the distance between the two points $P_1 = (1, 1)$ and $P_2 = (-2, 2)$.

1.1.21



The distance states that $d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. We have $P_1 = (x_1, y_1) = (1, 1)$ and $P_2 = (x_2, y_2) = (-2, 2)$,

so $d(P_1, P_2) = \sqrt{((-2)-(1))^2 + ((2)-(1))^2} = \sqrt{(-3)^2 + (-1)^2}$

$= \sqrt{9+1} = \sqrt{10}.$

$$\boxed{\sqrt{10}}$$

2. The midpoint of the line segment from P_1 to P_2 is $M = (-1, 4)$. If $P_1 = (-3, 6)$, what is P_2? WARNING: You are not *finding* the midpoint, you are *given* the midpoint and are finding point P_2.

1.1.55

The midpoint formula gives $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Here we have $P_1 = (x_1, y_1) = (-3, 6)$, $P_2 = (x_2, y_2)$, and $M = (-1, 4)$. So we have $(-1, 4) = \left(\frac{(-3) + x_2}{2}, \frac{(6) + y_2}{2}\right)$. So we need

$-1 = \frac{-3 + x_2}{2}$ or $-2 = -3 + x_2$ or $x_2 = 1$, $\boxed{P_2 = (x_2, y_2) = (1, 2).}$

and $4 = \frac{6 + y_2}{2}$ or $8 = 6 + y_2$ or $y_2 = 2$.

1.2.25

3. Find the intercepts of the graph of the equation $y = -x^2 + 4$.

For the x-intercept, we set $y=0$ and consider $0=-x^2+4$ or $x^2=4$ or $\sqrt{x^2}=\sqrt{4}$ or $|x|=2$ or $x=\pm 2$. So the x-intercepts are $(-2,0)$ and $(2,0)$. For the y-intercept we set $x=0$ and consider $y=-(0)^2+4=4$. So the y-intercept is $(0,4)$.

x-int. $(\pm 2, 0)$
y-int. $(0,4)$

1.2.11

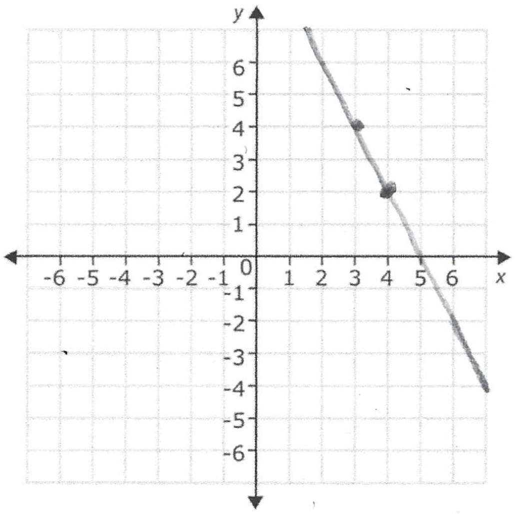
4. Test the equation $y = x^3$ for symmetry with respect to the x-axis, and the origin.

To test for symmetry with respect to the x-axis, replace y with $-y$ to get $(-y)=x^3$. This is not equivalent to the original equation since $(x,y)=(1,1)$ is on $y=x^3$ (because $1=(1)^3$) but is not on $-y=x^3$ (since $-(1)\neq(1)^3$). So the equation is not symmetric WRT the x-axis.
To test symmetry with respect to the origin, we replace x with $-x$ and replace y with $-y$ to get $(-y)=(-x)^3=-x^3$ or $y=x^3$ which __is__ the original equation. So the equation is symmetric WRT the origin.

NOT WRT x-axis,
symmetric WRT origin.

1,3,18

5. Plot the pair of points $(4, 2)$ and $(3, 4)$ and determine the slope of the line containing them. Graph the line.



The slope is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. With $(x_1, y_1) = (4,2)$ and $(x_2, y_2) = (3,4)$ we have a slope of

$$m = \frac{(4)-(2)}{(3)-(4)} = \frac{2}{-1} = -2.$$

$$\boxed{m = -2}$$

1,3,73

6. Find the equation of the line perpendicular to line $y = \frac{1}{2}x + 4$ such that the desired line contains the point $(1, -2)$. Express your answer in slope-intercept form.

The given line is in $y = m_1 x + b$ form and ~~so~~ has a slope of $m_1 = 1/2$. By the Criterion for Perpendicular Lines, the desired line has slope $m = -1/m_1 = \dfrac{-1}{(1/2)} = -2$. By the point slope formula, $y - y_1 = m(x - x_1)$, we have that the desired line is

$y - (-2) = (-2)(x - (1))$ or $y + 2 = -2(x-1) = -2x + 2$

or $y = -2x$.

$$\boxed{y = -2x}$$

1.4.27

7. Find the center and radius of the circle given by the equation $2(x-3)^2 + 2y^2 = 8$.

We put the equation in the standard form of a circle $(x-h)^2 + (y-k)^2 = r^2$ by dividing by 2 to get $(x-3)^2 + y^2 = 4$. So the center is $(h,k) = (3,0)$ and the radius is $r = \sqrt{4} = 2$.

Center $(3,0)$
radius $r=2$

2.1.38

8. Determine whether the equation $x^2 + y^2 = 1$ determines y as a function of x. Explain your answer.

We recognize $x^2+y^2=1$ as a circle with center $(0,0)$ and radius 1. So the graph violates the vertical line test. In particular, for $x=0$ we have $(0)^2+y^2=1$ or $y^2=1$ or $\sqrt{y^2}=\sqrt{1}$ or $|y|=1$ or $y=\pm 1$. So y is NOT a function of x since there is not a UNIQUE y associated with $x=0$.

y is NOT a function of x

Chapter 2
Review #9

9. Find the domain of the function $f(x) = \dfrac{x}{x^2 - 9}$. Express your answer in interval notation.

We cannot take square roots of negatives nor divide by 0. There are no square roots but there is division. So we have BAD x-values if $x^2 - 9 = 0$. That is, $(x+3)(x-3) = 0$. So we cannot have $x = -3$ or $x = 3$. So the domain is all real numbers except -3 and 3. In interval notation the domain is

$$\boxed{(-\infty,-3)\cup(-3,3)\cup(3,\infty)}$$