PRECALCULUS 1 (ALGEBRA) - TEST 2

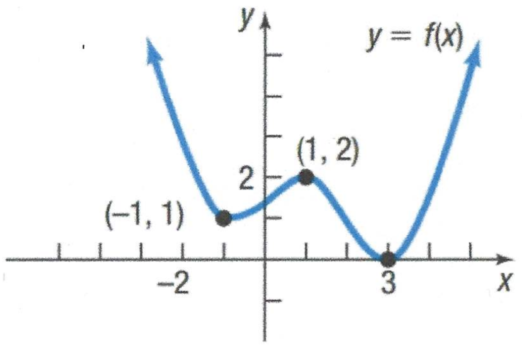
Fall 2021

NAME KEY STUDENT NUMBER ____________

SHOW ALL WORK!!! Be clear and convince me that you understand what you are doing. Use equal signs when quantities are equal (and do not use them when things are not equal). **Write in complete sentences** and explain your answer. Put your final answer in the box, when provided. Each numbered problem is worth 15 points. This test is two-sided and there are 9 questions. **No calculators, put away your cell phone and smart watch, and set the computer monitor face-down on the CPU!**

Example 2,3,4

1. Consider the graph of function f below. (a) At what numbers x, if any, does f have a local maximum? List the local maximum value(s). (b) At what numbers x, if any, does f have a local minimum? List the local minimum value(s). (c) Find the intervals on which f is increasing. Find the intervals on which f is decreasing.



(a) The point $(1,2)$ is higher than the other points on the graph near this point, so there is a local maximum at $x=1$ of 2.
(b) The points $(-1,1)$ and $(3,0)$ are lower than the other points on the graph near these points, so there is a local minimum at $x=-1$ of 1 and a local minimum at $x=3$ of 0.
(c) The function is increasing where it is going "uphill" as read from left to right, so f is increasing on $(-1,1)\cup(3,\infty)$.
The function is decreasing where it is going "downhill" as read from left to right, so f is decreasing on $(-\infty,-1)\cup(1,3)$.

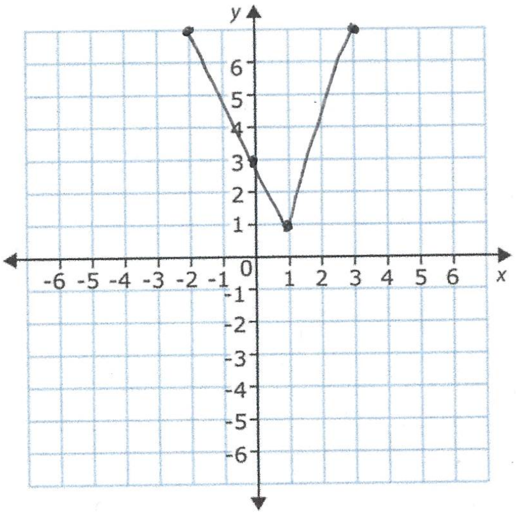
(a) local max at $x=1$ of 2
(b) local min at $x=-1$ of 1 and at $x=3$ of 0.
(c) INC on $(-1,1)\cup(3,\infty)$
DEC on $(-\infty,-1)\cup(1,3)$.

Exercise 2.4.33

2. Consider $f(x) = \begin{cases} -2x+3 & \text{if } x < 1 \\ 3x-2 & \text{if } x \geq 1. \end{cases}$ (a) Find the domain. (b) Locate intercepts. (c) Graph.

(d) Based on the graph, find the range.

(c)



(a) f is defined for all x with either $x<1$ or $x \geq 1$, so the domain is $\mathbb{R} = (-\infty, \infty)$.

(b) For the y-intercept, we set $x=0$ and get $f(0) = -2(0)+3 = 3$. So the y-intercept is 3.

For the x-intercept, we set each piece of f equal to 0 and consider $-2x+3=0$ or $2x=3$ or $x=3/2$. But for $x=3/2$, we do not use $f(x)=-2x+3$, so this does not give an x-intercept. Next, set $3x-2=0$ or $3x=2$ or $x=2/3$. But for $x=2/3$, we do not use $f(x)=3x-2$, so this does not give a x-intercept. So there are no x-intercepts.

(c) To graph, we plot two points for each linear piece of f. We have $f(-2)=-2(-2)+3=7$ and $f(0)=-2(0)+3=3$. Also $f(1)=3(1)-2=1$ and $f(3)=3(3)-2=7$.

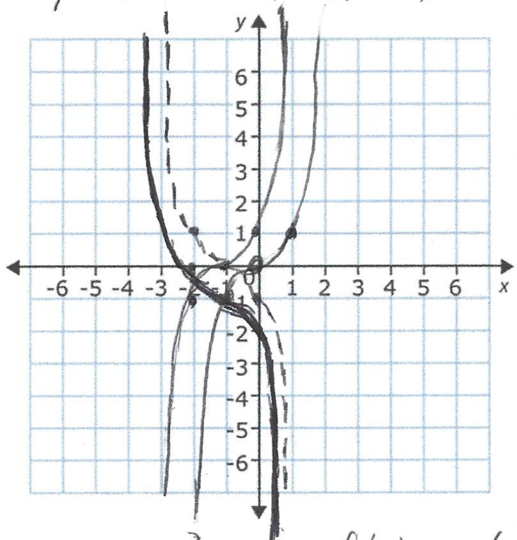
(d) Based on the graph, f takes on every value at or above $y=1$. So the range is $[1,\infty)$.

(a) Domain is $\mathbb{R} = (-\infty, \infty)$
(b) y-intercept is 3, no x-intercept
(d) Range is $[1, \infty)$

Exercise
2.5.55

3. Graph $f(x) = -(x+1)^3 - 1$ using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = x^3$ and show all steps. Be sure to show at least three key points. Explain the transformations you are using.

$y = -(x+1)^3$ $\quad$ $y = (x-1)^3$



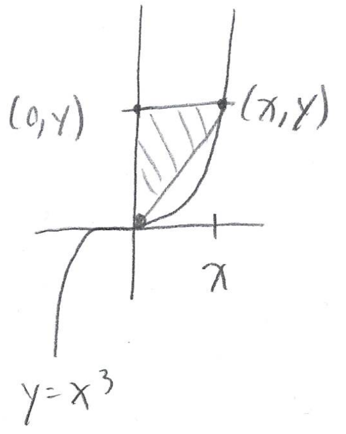
$y = x^3$ $\quad$ $y = f(x) = -(x+1)^3 - 1$

We start with $y = x^3$ and the key points $(-1,-1)$, $(0,0)$, $(1,1)$. Next, we translate to the left by 1 unit; by replacing x with $x-h$ we $h=-1$. This gives $y = (x-(-1))^3 = (x+1)^3$. Next, we reflect the graph about the x-axis by consider $y = -(x+1)^3$. Finally, we translate vertically down by 1 unit and consider $y = f(x) = -(x+1)^3 - 1$.
So we start with $y = x^3$, ① translate to the left 1 unit, ② reflect about the x-axis, and ③ translate vertically down 1 unit. This is given above.

Exercise 2.6.4

4. A right triangle has one vertex on the graph of $y = x^3$, $x > 0$, at (x, y), another at the origin, and the third on the positive y-axis at $(0, y)$. Express the area A of the triangle as a function of x.

The graph is

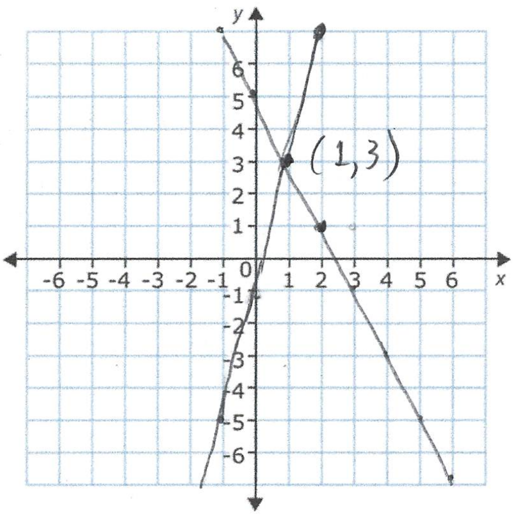


The area of a triangle is $\frac{1}{2}$(base)(height). The base of the triangle is x and then height is y. So the area is $A = \frac{1}{2}xy$. Since point (x,y) is on the graph of $y=x^3$. Then $A = \frac{1}{2}x(x^3) = \frac{1}{2}x^4$.

$$\boxed{A(x) = \frac{1}{2}x^4}$$

5. Suppose that $f(x) = 4x - 1$ and $g(x) = -2x + 5$. (a) Solve $f(x) = g(x)$. (b) Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution to the equation $f(x) = g(x)$.

Exercise 3.1.29



(a) For $f(x)=g(x)$, we consider $4x-1=-2x+5$ or $6x=6$ or $x=1$. When $x=1$, $f(1)=g(1)=4(1)-1=3$.

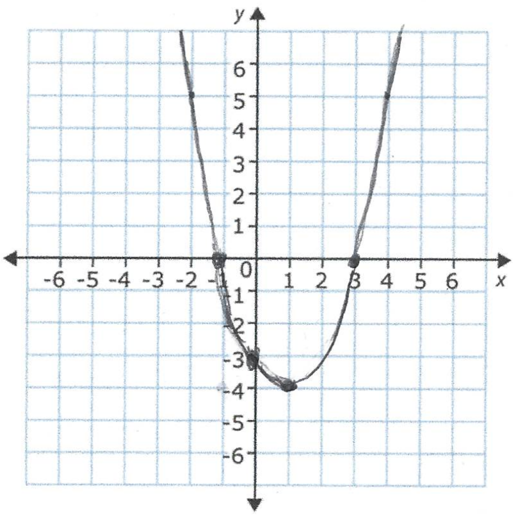
(b) We know the point $(1,3)$ lies on both lines. So we use slopes to find second points. For f, $m=4$ and we take $m=4=\Delta y/\Delta x$ where $\Delta y=4$ and $\Delta x=1$, and we get the second point $(x+\Delta x, y+\Delta y) = ((1)+(1),(3)+(4)) = (2,7)$.

For g, $m=-2=\Delta y/\Delta x$ and we take $\Delta y=-2$ and $\Delta x=1$, and we get the second point $(x+\Delta x, y+\Delta y) = ((1)+(1),(3)+(-2)) = (2,1)$.

$\boxed{(a)\ x=1}$

Exercise 3.3.48

6. Consider $f(x) = x^2 - 2x - 3$. (a) Find the vertex and axis of symmetry and determine whether the graph is concave up or concave down. (b) Find the y-intercept and the x-intercepts, if any. (c) Graph the function.



(a) For $f(x) = x^2 - 2x - 3$ we have $a = 1$, $b = -2$, $c = -3$. The vertex has x-coordinate $-b/(2a) = -(-2)/(2(1))$. The y-coordinate of the vertex is $f(1) = (1)^2 - 2(1) - 3 = -4$. The axis of symmetry is the line $x = -b/(2a) = 1$.

(b) For the y-intercept, set $x = 0$ to get $f(0) = (0)^2 - 2(0) - 3 = -3$. For the x-intercept, set $y = f(x) = 0$ to get $x^2 - 2x - 3 = 0$ or $(x-3)(x+1) = 0$ or $x = -1$ and $x = 3$. So the x-intercepts are -1 and 3.

(a) vertex $(1, -4)$,
axis $x = 1$
(b) y-intercept is -3
x-intercepts are -1 and 3