PRECALCULUS 1 (ALGEBRA) - TEST 3

Fall 2021

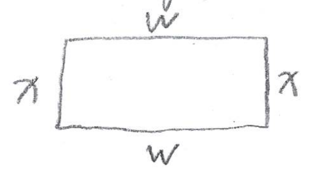
NAME KEY STUDENT NUMBER ____________

SHOW ALL WORK!!! Be clear and convince me that you understand what you are doing. Use equal signs when quantities are equal (and do not use them when things are not equal). **Write in complete sentences** and **explain your answers.** Put your final answer in the box, when provided. Each numbered problem is worth 15 points. This test is two-sided and there are 6 questions. **No calculators, put away your cell phone and smart watch, and set the computer monitor face-down on the CPU!**

1. David has 400 yards of fencing and wishes to enclose a rectangular area. (a) Express the area A of the rectangle as a function of the width w of the rectangle. (b) For what value of w is the area largest?

Problem 3, 4, 7(a, b)

With w as the width of the field and x as the height, the field is:



Since the perimeter of the rectangle is $2x+2w$ and David has 400 yards of fencing, then $2x+2w=400$ or $2x=400-2w$ or $x=200-w$

(a) The area of the rectangle is $A=wx$ or as a function of w, $A(w)=w(200-w)=200w-w^2$.

(b) With quadratic function $A(w)=200w-w^2$ in aw^2+bw+c form, we have $a=-1$, $b=200$, and $c=0$. Since $a=-1<0$ then $A(w)$ is concave down and has an absolute maximum at the vertex where $w=-b/(2a)$. So the maximum occurs at

$w=-(200)/(2(-1))$

$=100$ yards.

$\boxed{\begin{array}{l}(a)\ A(w)=200w-w^2 \\ (b)\ w=100 \text{ yards}\end{array}}$

Problem 4.1.49

2. Find a polynomial of degree $n = 3$ with the real zeros $r_1 = -2$, $r_2 = 3$, and $r_3 = 5$, whose graph contains the point $(2, 36)$.

Since $r_1 = -2$ is a zero of the polynomial p then $(x-r_1)^{m_1} = (x-(-2))^{m_1} = (x+2)^{m_1}$ is a factor of p for some integer $m_1 \geq 1$. Similarly $(x-r_2)^{m_2} = (x-3)^{m_2}$ is a factor of p for some integer $m_2 \geq 1$, and $(x-r_3)^{m_3} = (x-5)^{m_3}$ is a factor of p for some integer $m_3 \geq 1$. Therefore $(x+2)^{m_1}(x-3)^{m_2}(x-5)^{m_3}$ is a factor of p. Since this factor is of degree $m_1+m_2+m_3$ where $3 \leq m_1+m_2+m_3$ and p is of degree 3, then we must have $m_1=m_2=m_3=1$ and there are no other polynomial factors of p of positive degree. So p must be of the form
$p(x) = a_3(x+2)(x-3)(x-5)$. Since $(2,36)$ is a point on the graph of p then
$p(2) = 36$ and so

$$p(2) = a_3((2)+2)((2)-3)((2)-5)$$
$$= 12a_3 = 36 \text{ and so}$$

$a_3 = 3$. Therefore $p(x) = 3(x+2)(x-3)(x+5)$.

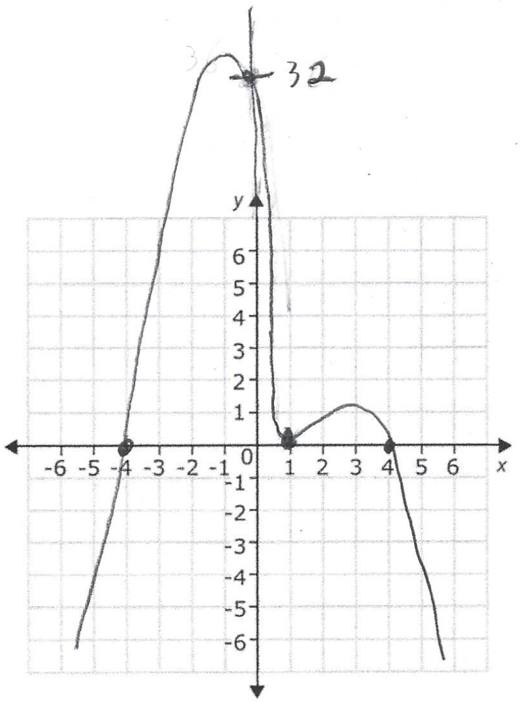
$$\boxed{p(x) = 3(x+2)(x-3)(x+5)}$$

Problem 4,2,17

3. Consider $f(x) = -2(x-1)^2(x^2-16)$. (a) Find the x-intercepts and y-intercept (which will be off the scale given below). (b) Determine whether the graph crosses or touches the x-axis at the x-intercepts. Graph.

(a) To find the y-intercept, set $x=0$ and consider $f(0) = -2(-1)^2(-16) = 32$
So the y-intercept is 32. For the x-intercept, set
$y = f(x) = 0$ and consider $-2(x-1)^2(x^2-16) = 0$ or
$-2(x-1)^2(x-4)(x+4) = 0$ and the x-intercepts are -4, 1, and 4.

(b) Since the factor $x-1$ is of even multiplicity then the graph touches the x-axis at $x=1$. Since the factors $(x-4)$ and $(x+4)$ are of odd multiplicity then the graph crosses the x-axis at -4 and 4.



(a) y-intercept: 32
x-intercepts: -4, 1, 4
(b) touches x-axis at 1
crosses x-axis at -4 and 4

Problem 4.3.48

4. Find the vertical, horizontal, and oblique asymptotes, if any, of rational function
$$G(x) = \frac{x^3+1}{x^2-5x-14}.$$

We have $G(x) = \frac{p(x)}{q(x)}$ where p is of degree $n=3$ and q is of degree $m=2$. Since $n = m+1$ then G has an oblique asymptote. We find it by long division:

$$\begin{array}{r l}
 & x+5 \\
x^2-5x-14 \,\big)\!\! & \overline{x^3 \qquad\qquad\quad +1} \\
 & \underline{-(x^3-5x^2-14x)} \\
 & 5x^2+14x+1 \\
 & \underline{-(5x^2-25x-70)} \\
 & 39x+71
\end{array}$$

So the oblique asymptote is $y = x+5$. Since G has an oblique asymptote then it does not have a horizontal asymptote. Since the denominator is $q(x) = x^2-5x-14$ $= (x+2)(x-7)$ then G has vertical asymptotes of $x=-2$ and $x=7$.

Vertical: $x=-2$, $x=7$

Horizontal: None

Oblique: $y=x+5$

Example 4.4.1

5, 6. Consider $R(x) = \dfrac{x-1}{x^2-4}$. (a) Find the horizontal and vertical asymptotes. (b) Find where the graph of R is above or below the x-axis. (c) Graph $y = R(x)$. (d) Solve the inequality $\dfrac{x-1}{x^2-4} \geq 0$.

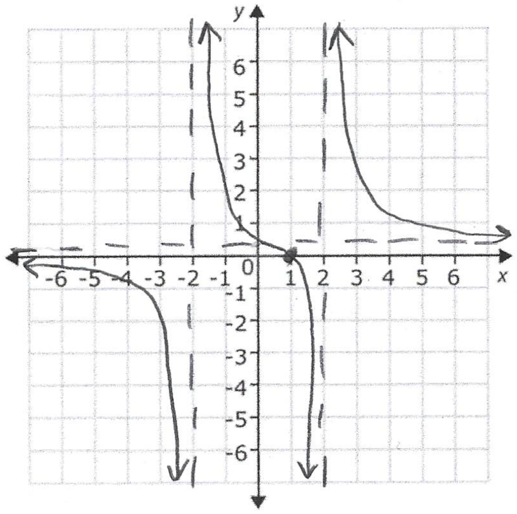
(a) Since the numerator is of degree $n=1$ and the denominator is of degree $m=2$ and $m>n$ then $y=0$ is the horizontal asymptote. Since the denominator is $x^2-4=(x+2)(x-2)$ then the vertical asymptotes are $x=-2$ and $x=2$.

(b) We use the zeros of the numerator and denominator to determine intervals on which R has the same sign:

Interval	$(-\infty,-2)$	$(-2,1)$	$(1,2)$	$(2,\infty)$
Test Value c	-3	0	$3/2$	3
$R(c)$	$(-4)/(5)$	$(-1)/(-4)$	$(1/2)/(-7/4)$	$(2)/(5)$
Conclusion	R negative	R positive	R negative	R positive

R is above the x-axis on $(-2,1)\cup(2,\infty)$.
R is below the x-axis on $(-\infty,-2)\cup(1,2)$.

(c)



(d) From the graph or the table, we have $R(x)=\dfrac{x-1}{x^2-4}\geq 0$ for x in $(-2,1]\cup(2,\infty)$.

(a) H.A. $y=0$, V.A. $x=-2$, $x=2$
(b) Above x-axis $(-2,1)\cup(2,\infty)$
Below x-axis $(-\infty,-2)\cup(1,2)$
(d) $(-2,1]\cup(2,\infty)$.