PRECALCULUS 1 (ALGEBRA) - TEST 4

Fall 2021

NAME KEY STUDENT NUMBER ____________

SHOW ALL WORK!!! Be clear and convince me that you understand what you are doing. Use equal signs when quantities are equal (and do not use them when things are not equal). **Write in complete sentences** and **explain your answers**. Put your final answer in the box, when provided. Each numbered problem is worth 15 points. This test is two-sided and there are 7 questions. **No calculators, put away your cell phone and smart watch, and set the computer monitor face-down on the CPU!**

Exercise 4.6.18

1. Consider $f(x) = x^6 - 16x^4 + x^2 - 16$ and $x - c = x - (-4) = x + 4$. Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $x - c = x + 4$. Then use the Factor Theorem to determine whether $x - c$ is a factor of $f(x)$.

By the Remainder Theorem, if polynomial function f is divided by $x-c$ then the remainder is $f(c)$. So if we divide $f(x) = x^6 - 16x^4 + x^2 - 16$ by $x+4 = x-(-4)$ then the remainder is $f(-4) = (-4)^6 - 16(-4)^4 + (-4)^2 - 16$
$= 4^6 - 4^2 \cdot 4^4 + 4^2 - 16 = 4^6 - 4^6 + 16 - 16 = 0.$

By the Factor Theorem, $x-c$ is a factor of $f(x)$ if and only if $f(c) = 0$. With $c = -4$, we see that $f(-4) = 0$ and so $x+4$ is a factor of $f(x)$.

Remainder: 0
YES, $x+4$ is a factor

Exercise
5.1.33(a,b)

2. For $f(x) = \sqrt{x}$ and $g(x) = 2x + 5$ find the compositions and domains for $f \circ g$ and $g \circ f$.

We have $(f \circ g)(x) = f(g(x)) = f(2x+5) = \sqrt{2x+5}$.
For x in the domain, we need $2x+5 \geq 0$ or $2x \geq -5$
or $x \geq -5/2$. The domain is $[-5/2, \infty)$.

We have $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = 2\sqrt{x}+5$.
For x in the domain, we need $x \geq 0$ and the
domain is $[0, \infty)$.

$(f \circ g)(x) = \sqrt{2x+5}$, domain is $[-5/2, \infty)$.
$(g \circ f)(x) = 2\sqrt{x}+5$, domain is $[0, \infty)$.

Exercise 5.2.40

3. Consider $f(x) = (x-2)^2$ for $x \geq 2$ and $g(x) = \sqrt{x} + 2$. Verify that the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Give any values of x that need to be excluded from the domain of f and g.

First, $f(g(x)) = f(\sqrt{x}+2) = ((\sqrt{x}+2)-2)^2 = (\sqrt{x})^2$

$= x$ for $x \geq 0$.

So $f(g(x)) = x$ for all x in the domain of g.

Second, $g(f(x)) = g((x-2)^2) = \sqrt{(x-2)^2} + 2$

$= |x-2| + 2 = (x-2)+2$ since $x \geq 2$

$= x$ for $x \geq 2$.

So $g(f(x)) = x$ for all x in the domain of f.

Exercise 5.2.56(a)

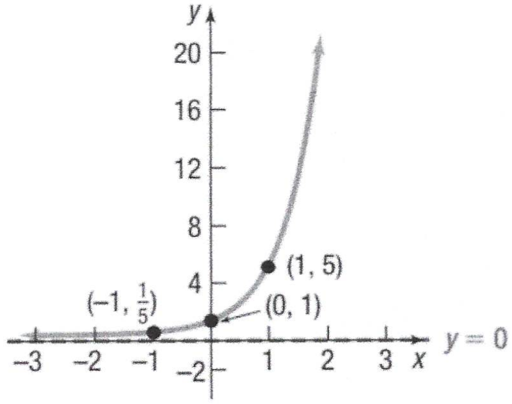
4. Consider $f(x) = x^3 + 1$ ~~for $x \geq 2$~~. Find its inverse f^{-1}. What is the domain and range of f^{-1}?

First, we set $y = f(x) = x^3 + 1$. Next, we interchange x and y to get $x = y^3 + 1$. Finally, we solve for y: $y^3 = x - 1$ or $\sqrt[3]{y^3} = \sqrt[3]{x-1}$ or $y = \sqrt[3]{x-1}$. So $f^{-1}(x) = \sqrt[3]{x-1}$. The domain of f^{-1} is $\mathbb{R} = (-\infty, \infty)$. The range of $f^{-1}(x)$ is the domain of f and hence is $\mathbb{R} = (-\infty, \infty)$.

$f^{-1}(x) = \sqrt[3]{x-1}$,
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

Exercise
5.3.92

5. Determine the exponential function:



We look for a function of the form $f(x) = Ca^x$. Since point $(0,1)$ is on the graph, then $f(0) = Ca^0 = C = 1$, and so $C = 1$. Therefore $f(x) = Ca^x = (1)a^x = a^x$. Next, since point $(1,5)$ is on the graph, then $f(1) = a^1 = a = 5$, and so $a = 5$. Therefore, $f(x) = 5^x$.

$$\boxed{f(x) = 5^x}$$

Exercise 5.3.73

6. (a) Solve for x: $3^{x^3} = 9^x$.

Since $9=3^2$ then $3^{x^3} = 9^x = (3^2)^x = 3^{2x}$. Since an exponential function is one-to-one then we have $x^3 = 2x$ or $x^3 - 2x = 0$ or $x(x^2-2)=0$ or $x(x-\sqrt{2})(x+\sqrt{2})=0$. So the solutions are $x=0$, $x=\sqrt{2}$, and $x=-\sqrt{2}$.

$x=0,\ x=\sqrt{2},\ x=-\sqrt{2}$

Exercise 5.4.96

(b) Solve for x: $\ln e^{-2x} = 8$.

Since $\ln(x) = \log_e(x)$, then $\ln(e^{-2x}) = \log_e(e^{-2x}) = 8$. By the definition of logarithm, this means $e^8 = (e^{-2x})$. Since an exponential function is one-to-one then we have $8 = -2x$ or $x = -4$.

$x=-4$

Exercise 5.5.93

7. Express y as a function of x. The constant C is a positive number: $\ln(y-3) = -4x + \ln(C)$.

We have $\ln(y-3) - \ln(C) = -4x$. Exponentiating we get

$$e^{\ln(y-3) - \ln(C)} = e^{-4x}$$

or $e^{\ln(y-3)} e^{-\ln(C)} = e^{-4x}$

or $e^{\ln(y-3)} (e^{\ln C})^{-1} = e^{-4x}$

or $(y-3)(C^{-1}) = e^{-4x}$ since the base e exponential is the inverse of the natural logarithm

or $y-3 = Ce^{-4x}$ or $y = 3 + Ce^{-4x}$.

$$\boxed{y = 3 + Ce^{-4x}}$$