

## Chapter 2. Graphs

### 2.1. Rectangular Coordinates

**Note.** In preparation for this section, you may need to review Sections R.2 and R.3.

**Note.** Recall that the real number line is represented by a line on which each point is associated with a real number. The *origin* of the number line is the point associated with the number 0. If we take two real number lines, one horizontal called the *x-axis* and the other vertical called the *y-axis*, and then intersect them at their origins, we have produced the *xy-plane* or the *Cartesian plane*. The point at which the lines intersect is called the *origin* of the plane. The *x-* and *y-*axes are also called the *coordinate axes*.

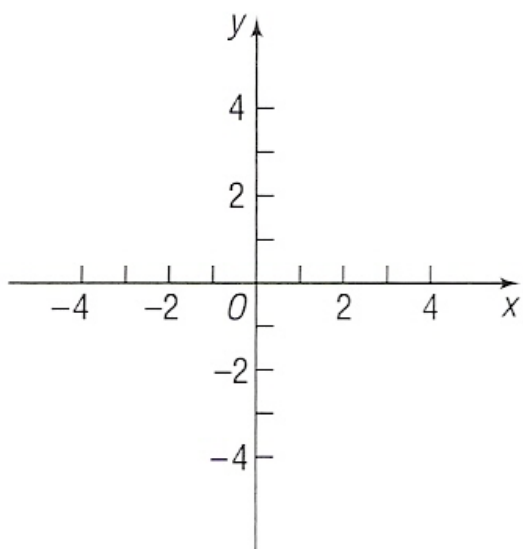


Figure 1 Page 158.

**Definition.** Any point  $P$  in the  $xy$ -plane can be located using an *ordered pair*  $(x, y)$  of real numbers. Let  $x$  denote the signed distance of  $P$  from the  $y$ -axis (*signed* in the sense that, if  $P$  is to the right of the  $y$ -axis, then  $x > 0$  and if  $P$  is the left of the  $y$ -axis, then  $x < 0$ ); and let  $y$  denote the signed distance of  $P$  from the  $x$ -axis. The ordered pair  $(x, y)$ , also called the *coordinates* of  $P$  then gives us enough information to locate the point  $P$  in the plane.

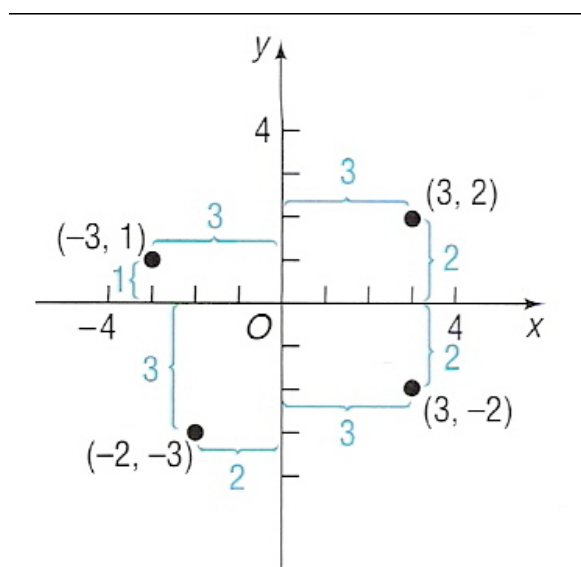


Figure 2 Page 158.

**Definition.** If  $(x, y)$  are the coordinates of a point  $P$ , then  $x$  is called the *x-coordinate*, or *abscissa*, of  $P$ , and  $y$  is the *y-coordinate*, or *ordinate*, of  $P$ . We identify the point  $P$  by its coordinates  $(x, y)$  by writing  $P = (x, y)$ , referring to it as “the point  $(x, y)$ ,” rather than “the point whose coordinates are  $(x, y)$ .”

**Definition.** The coordinate axes divide the  $xy$ -plane into four sections, called *quadrants*. In quadrant I, both the  $x$ -coordinate and the  $y$ -coordinate of all points are positive; in quadrant II,  $x$  is negative and  $y$  is positive; in quadrant III, both  $x$  and  $y$  are negative; and in quadrant IV,  $x$  is positive and  $y$  is negative. Points on the coordinate axes belong to no quadrant.

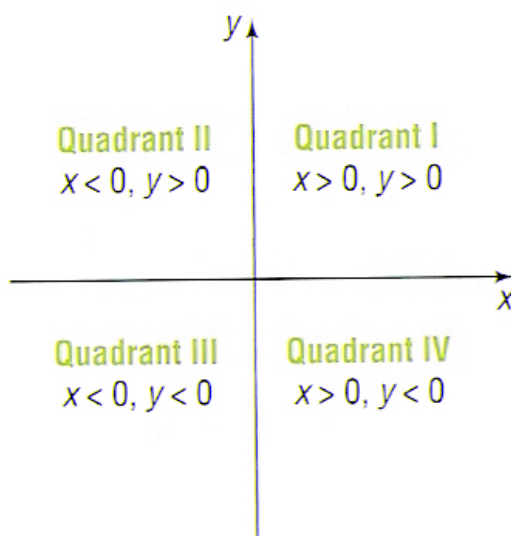


Figure 3 Page 159.

**Example.** Page 163 number 12.

**Note.** We can use the Pythagorean theorem to prove (see page 160 for details):

**Theorem.** The distance between two points  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$  is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

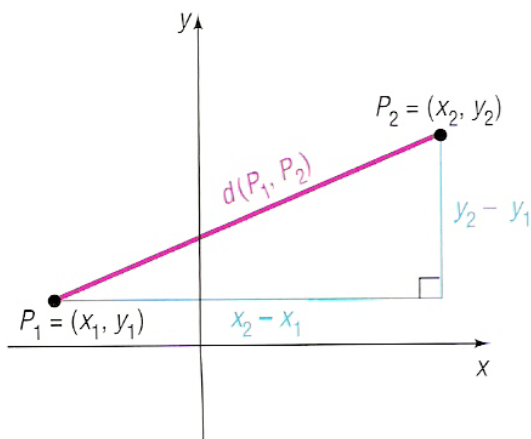


Figure 6 Page 160.

**Example.** Page 163 numbers 20, 24, and 32.

**Definition.** The *midpoint*  $M = (x, y)$  of the line segment from  $P_1 = (x_1, y_1)$  to  $P_2 = (x_2, y_2)$  is

$$M = (x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

**Example.** Page 164 numbers 44, 53, and 66.