Chapter 4. Polynomial and Rational Functions

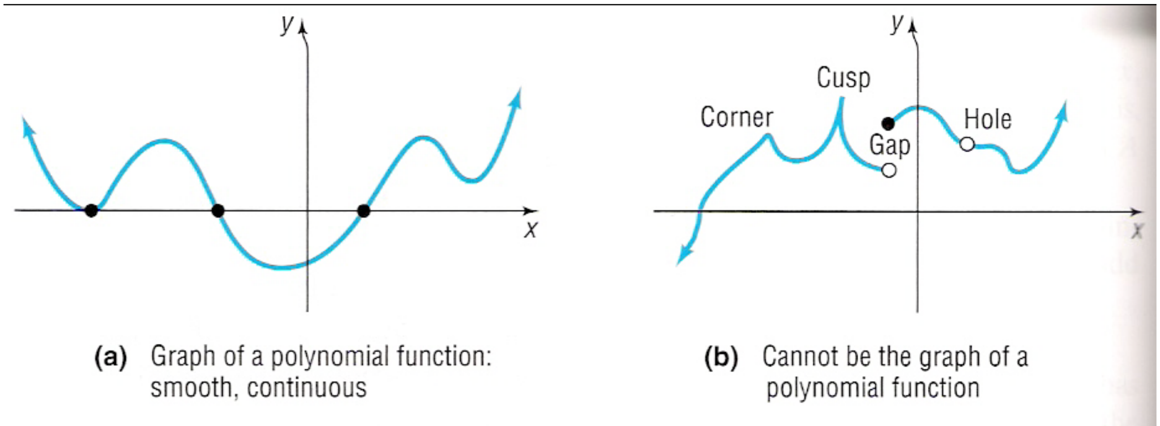
4.2. Polynomial Functions

Note. In preparation for this section, you may need to review Appendix A Section R.4, Section 2.2, Section 3.3, and Section 3.5.

Definition. A *polynomial function* is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ where n is a nonnegative integer. The domain of a polynomial function is the set of all real numbers. The *degree* of a polynomial is the largest power of x in the polynomial.

Example. Page 327 number 22.

Note. In calculus, you will see that a polynomial function is *smooth* (i.e. its graph has no sharp corners or cusps) and *continuous* (i.e. its graph has no gaps or holes and can be drawn without lifting pencil from paper).



Page 314 Figure 21

Definition. A *power function of degree* n is a function of the form $f(x) = ax^n$ where $a \neq 0$ and $n > 0$ is an integer.

Note. Properties of Power Functions, $y = x^n$, n an Even Integer

1. The graph is symmetric with respect to the y-axis.

2. The domain is the set of all real numbers, $x \in (-\infty, \infty)$. The range is the set of nonnegative real numbers, $y \in [0, \infty)$.

3. The graph always contains the points $(0, 0)$, $(1, 1)$, and $(-1, 1)$.

4. As the exponent n increases in magnitude, the graph becomes more vertical when $x < -1$ or $x > 1$; but for x near the origin (that is, $-1 < x < 1$), the graph tends to flatten out and lie closer to the x-axis.

Note. Properties of Power Functions, $y = x^n$, n an Odd Integer

1. The graph is symmetric with respect to the origin.

2. The domain and range are the set of all real numbers, $(-\infty, \infty)$.

3. The graph always contains the points $(0, 0)$, $(1, 1)$, and $(-1, -1)$.

4. As the exponent n increases in magnitude, the graph becomes more vertical when $x < -1$ or $x > 1$; but for x near the origin (that is, $-1 < x < 1$), the graph tends to flatten out and lie closer to the x-axis.

Example. Page 327 number 32.

Definition. If f is a polynomial function and r is a real number for which $f(r) = 0$, then r is called a *zero* of f, or *root* of f.

Note. If r is a zero of f, then

(a) r is an x-intercept of the graph of f, and

(b) $(x - r)$ is a factor of f.

Definition. If $(x - r)^m$ is a factor of polynomial f and $(x - r)^{m+1}$ is not a factor of f, then r is a zero of *multiplicity* m of f.

Note. If r is a zero of polynomial function f of even multiplicity, then f does not change sign from one side to the other of r. So the graph of f touches the x-axis at $x = r$. If r is a zero of polynomial function f of odd multiplicity, then f changes sign from one side to the other of r. So the graph of f crosses the x-axis at $x = r$.

Example. Page 327 number 68ab.

Definition. The points at which a graph changes direction (from increasing to decreasing, or from decreasing to increasing) is called a *turning point* of the graph. At these points, a continuous function will have its local maxima and local minima.

Theorem. If f is a polynomial function of degree n, then f has at most $n - 1$ turning points.

Theorem. For large values of x, either positive and negative, the graph of the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ resembles the graph of the power function $y = a_n x^n$ and this function is called the *end behavior model* of f.

Example. Page 327 number 68cd.

Note. If we know the zeros of a polynomial, then we can use "test values" to determine where the polynomial is positive and where it is negative.

Example. Page 327 number 68ef.