Chapter 1. Graphs

Section 1.1. The Distance and Midpoint Formulas

Note. In this section we introduce the Cartesian plane and rectangular coordinates, the distance formula between two points in the Cartesian plane, and the midpoint formula for the location of a point midway between two points in the Cartesian plane.

Definition. Recall that the real *number line* is represented by a line on which each point is associated with a real number. The *origin* of the number line is the point associated with the number 0. If we take two real number lines, one horizontal called the x-axis and the other vertical called the y-axis, and then intersect them at their origins, we have produced the *xy-plane* or the *Cartesian plane.* The point at which the lines intersect is called the *origin* of the plane. The x- and y-axes are also called the *coordinate axes.*

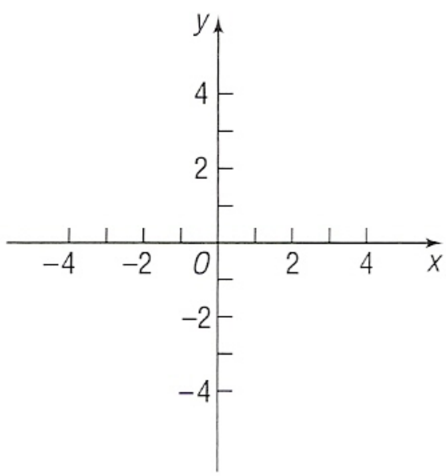

Figure 1 Page 2

Definition. Any point P in the xy-plane can be located using an *ordered pair* (x, y) of real numbers. Let x denote the *signed* distance of P from the y-axis (signed in the sense that, if P is to the right of the y-axis, then $x > 0$ and if P is the left of the y-axis, then $x < 0$) and let y denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the *coordinates* of P, then gives us enough information to locate the point P in the plane.

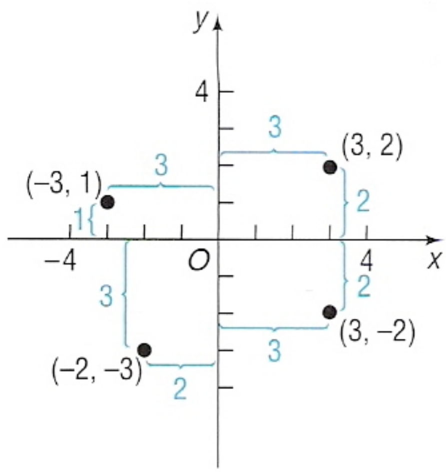

Figure 2 Page 2

Definition. If (x, y) are the coordinates of a point P, then x is called the *x-coordinate*, or *abscissa*, of P and y is the *y-coordinate*, or *ordinate*, of P. We identify the point P by its coordinates (x, y) by writing $P = (x, y)$, referring to it as "the point (x, y)," rather than "the point whose coordinates are (x, y)."

Definition. The coordinate axes divide the xy-plane into four sections, called *quadrants*. In quadrant I, both the x-coordinate and the y-coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x

and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.

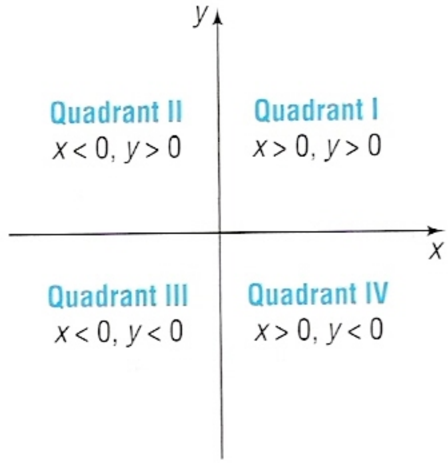

Figure 2 Page 3

Examples. Page 7 numbers 16 and 18.

Theorem 1.1.A. The Distance Formula. The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Examples. Page 7 numbers 26 and 34.

Definition. Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be the endpoints of a line segment and let $M = (x, y)$ be the point on the line segment that is the same distance from P_1 as it is from P_2. Point M is the *midpoint* of the line segment.

Theorem 1.1.B. The Midpoint Formula. The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Example. Page 7 numbers 40 and 44.

Example. Page 8 numbers 56 and 68.

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