Section 1.2. Graphs of Equations in Two Variables; Intercepts; Symmetry

Note. In this section we graph equations by plotting points, find intercepts from a graph, find intercepts from an equation, and test equations for symmetries with respect to axes and the origin.

Definition. An *equation in two variables*, say x and y, is a statement in which two expressions involving x and y are equal. Any values of x and y that result in a true statement are said to *satisfy* the equation. The *graph of an equation* in two variables x and y is the set of all points (x, y) in the xy-plane where x and y satisfy the equation.

Example. Page 17 number 18.

Definition. The points, if any, at which a graph crosses or touches the coordinate axes are the *intercepts* of the graph. The x-coordinate of a point at which the graph crosses or touches the x-axis is an *x-intercept* and the y-coordinate of a point at which the graph crosses or touches the y-axis is a *y-intercept*.

Note. The procedure for finding intercepts is:

To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x.

To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y.

Example. Page 17 number 20. HINT: The graph is a line. In general, plotting points is an awful way to graph a function, but if you know the general shape of the function and you plot special points, *then* this is a good approach.

Definition. We consider three different types of symmetries of graphs:

A graph is said to be *symmetric with respect to the x-axis* if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

A graph is said to be *symmetric with respect to the y-axis* if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

A graph is said to by *symmetric with respect to the origin* if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

Examples. Page 17 numbers 48 and 56.

Note. To test the graph of an equation for symmetry with respect to the

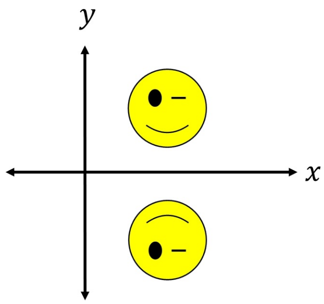
x-Axis Replace y by $-y$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.

y-Axis Replace x by $-x$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the y-axis.

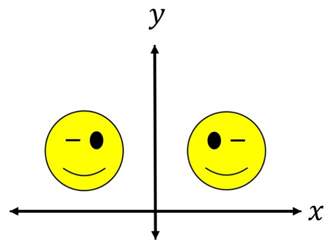
Origin Replace x by $-x$ and y by $-y$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the origin.

Recall that two equations are *equivalent* if they have the same solution set (see Appendix A.6. Solving Equations).

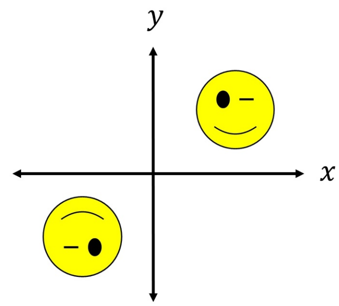
Note. We illustrate symmetries with a familiar graph:



Symmetry with respect to the x-axis



Symmetry with respect to the y-axis



Symmetry with respect to the origin

Example. Page 18 number 72.

Note. We now loosely discuss the graphs of three special equations.

Examples. Page 11 Example 3 and Page 15 Example 10, and Page 16 Example 12.

Example. Page 18 number 84.

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