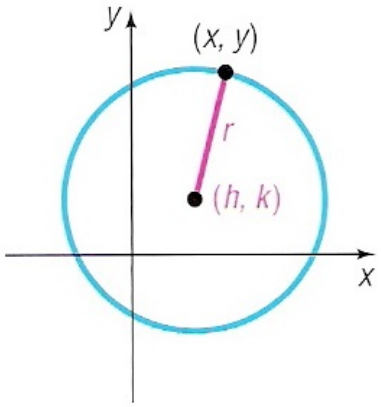
Section 1.4. Circles

Note. In this section we write the standard form of the equation of a circle, graph circles, and work with the general form of the equation of a circle.

Definition. A *circle* is a set of points in the xy-plane that are a fixed distance r from a fixed point (h, k). The fixed distance r is called the *radius*, and the fixed point (h, k) is called the *center* of the circle.



Page 34 Figure 49.

Note. If (x, y) is a point on the circle described above, then the distance from (x, y) to (h, k) is r and so by the distance formula

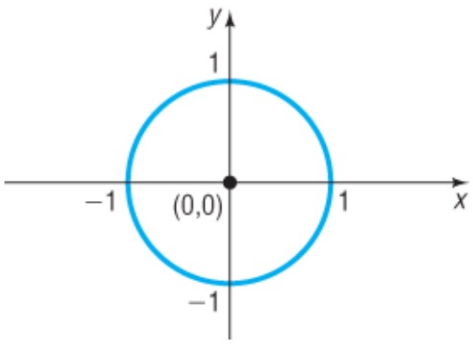
$$\sqrt{(x-h)^2+(y-k)^2} = r.$$

Squaring both sides (since r is a distance, $r > 0$ and we do not introduce extraneous roots): $(x-h)^2+(y-k)^2 = r^2$.

Definition. The *standard form of an equation of a circle* with radius r and center (h, k) is $(x-h)^2 + (y-k)^2 = r^2$.

Theorem 1.4.A. The standard form of an equation of a circle of radius r with center at the origin $(0, 0)$ is $x^2 + y^2 = r^2$.

Definition. If $r = 1$, the circle whose center is at the origin is called the *unit circle* and has the equation $x^2 + y^2 = 1$.



Page 35 Figure 50.

Examples. Page 38 numbers 18, 24, and 36.

Note. If we multiply the standard form of a circle out, we get an equation of the form $x^2 + y^2 + ax + by + c = 0$. However, not all equations of this form represent circles (for example, if $a = b = c = 0$).

Definition. When its graph is a circle, the equation

$$x^2 + y^2 + ax + by + c = 0$$

is the *general form of the equation of a circle.*

Examples. Page 38 number 40, 52, 54.

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