Section 2.3. Properties of Functions

Note. In this section we define even/odd functions and identify them, define increasing/decreasing functions, use a graph to find local maxima and minima, use a graph to find absolute maximum and minimum, and the find average rate of change of a function on an interval.

Definition. A function f is *even* if, for every number x in its domain, the number $-x$ is also in the domain and $f(-x) = f(x)$. Function f is *odd* if for every x in its domain, $-x$ is also in the domain and $f(-x) = -f(x)$.

Note. We can recognize even and oddness from the graph of a function with the following theorem.

Theorem 2.3.A. A function is even if and only if its graph is symmetric with respect to the y-axis. A function is odd if and only if its graph is symmetric with respect to the origin.

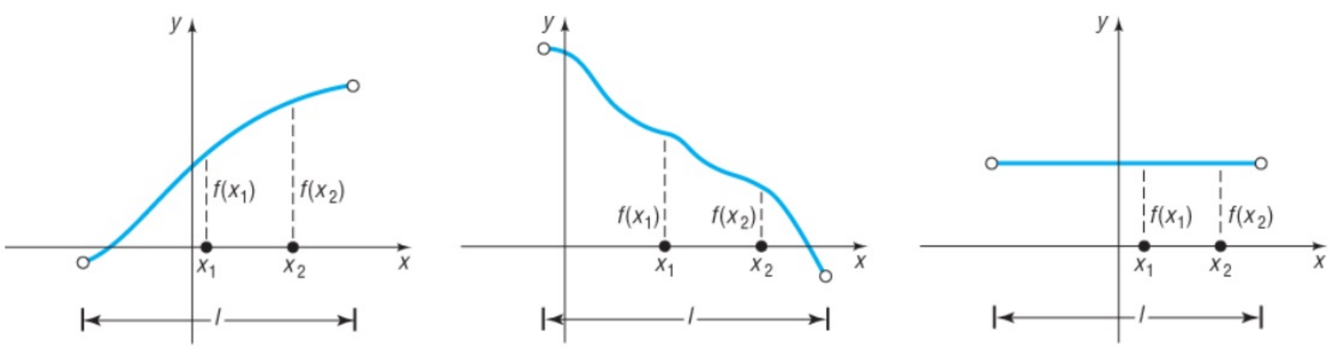
Examples. Page 78 numbers 26(a),(b),(d) and 28(a)(b)(d).

Definition. A function f is

(1) *increasing* on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$,

(2) *decreasing* on open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$, and

(3) *constant* on interval I if, for all choices of x in I, the values of $f(x)$ are equal.



(a) For $x_1 < x_2$ in I, $f(x_1) < f(x_2)$; f is increasing on I.

(b) For $x_1 < x_2$ in I, $f(x_1) > f(x_2)$; f is decreasing on I.

(c) For all x in I, the values of f are equal; f is constant on I.

Page 72 Figure 19

Examples. Page 78 numbers 26(c) and 28(c).

Definition. Let f be a function defined on some interval I. Function f has a *local maximum at* c if there is an open interval containing c so that, for all $x \neq c$ in I, $f(x) < f(c)$. We call $f(c)$ the *local maximum* of f. Function f has a *local minimum at* c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) > f(c)$. We call $f(c)$ the *local minimum* of f.

Note. The idea here is that, at a local maximum, f is greater than it is at "nearby" values. At a local minimum, f is lesser than it is at "nearby" values. The function may get greater or lesser at other values, but *near* c these properties hold. The "nearness" determines how big (or small) interval I must be. These values (together called "local extrema") are easy to find *if* you have a graph of $y = f(x)$.

Example. Page 79 number 34.

Definition. Let f be a function defined on some interval I. If there is a number u in I for which $f(x) \leq f(u)$ for all x in I, then f has an *absolute maximum at* u, and the number $f(u)$ is the *absolute maximum of* f *on* I. If there is a number v in I for which $f(x) \geq f(v)$ for all x in I, then f has an *absolute minimum at* v, and the number $f(v)$ is the *absolute minimum of* f *on* I. The absolute maximum and minimum of a function f are sometimes called the *extreme values* of f on I.

Examples. Page 79 numbers 50 and 52.

Note. As we just saw, not all functions have an absolute maximum or an absolute minimum. The following theorem, which you will see in Calculus 1 (see Theorem 1 in my online notes for 4.1. Extreme Values of Functions).

Theorem 2.3.B. Extreme Value Theorem. If f is a continuous function whose domain is a closed interval $[a, b]$, then f has an absolute maximum and an absolute minimum on $[a, b]$.

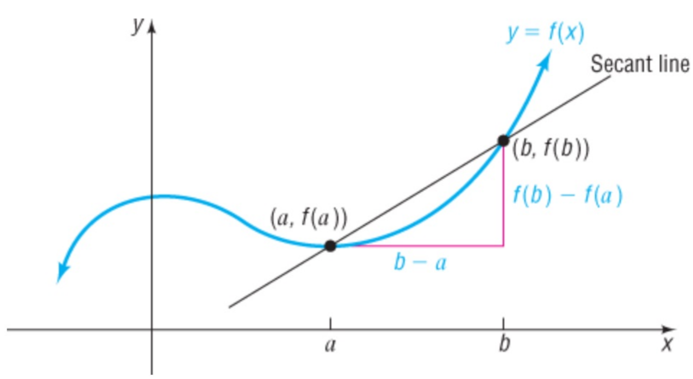
Definition. If a and b are in the domain of a function $y = f(x)$, the *average rate of change of f* from a to b is defined as

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

where $a \neq b$. In calculus, this expression is called a *difference quotient.*

Definition. Let $y = f(x)$ be a function with a and b in its domain. The line containing the points $(a, f(a))$ and $(b, f(b))$ is the *secant line.*

Note. The slope of the secant line as just defined is $m_{\text{sec}} = \dfrac{f(b) - f(a)}{b - a}$. This immediately gives the theorem stated below.



Page 77 Figure 26

Theorem 2.3.C. Slope of a Secant Line. The average rate of change of a function from a to b equals the slope of the secant line containing the two points $(a, f(a))$ and $(b, f(b))$ on its graph.

Example. Page 80 number 72.

Example. Page 81 number 82.

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