

Section 2.5. Graphing Techniques: Transformations

Note. In this section we graph functions using the following transformations: vertical and horizontal shifts, compressions and stretches, and reflections about axes.

Definition. If a real number k is added to a function $y = f(x)$, the graph of the new function $y = f(x) + k$ is the graph of f *shifted vertically up* (if $k > 0$) or *down* (if $k < 0$).

Definition. If the argument x of a function f is replaced by $x - h$, h a real number, the graph of the new function $y = f(x - h)$ is the graph of f *shifted horizontally left* (if $h < 0$) or *right* (if $h > 0$).

Example. Find a formula for the function whose graph is the graph of $y = \sqrt[3]{x}$ but is **(a)** shifted to the right 4 units, **(b)** shifted to the left 4 units, **(c)** shifted up 4 units, **(d)** shifted down 4 units.

Definition. When the right side of a function $y = f(x)$ is multiplied by a positive number a , the graph of the new function $y = af(x)$ is obtained by multiplying each y -coordinate of $y = f(x)$ by a . A *vertical compression* results if $0 < a < 1$ and a *vertical stretch* occurs if $a > 1$.

Example. Find a formula for the function whose graph is the graph of $y = \sqrt[3]{x}$ but is **(a)** vertically stretched where $a = 4$, **(b)** vertically compressed where $a = 1/4$.

Definition. If the argument x of a function $y = f(x)$ is multiplied by a positive number a , the graph of the new function $y = f(ax)$ is obtained by multiplying each x -coordinate of $y = f(x)$ by $1/a$. A *horizontal compression* results if $a > 1$, and a *horizontal stretch* occurs if $0 < a < 1$.

Example. Find a formula for the function whose graph is the graph of $y = \sqrt[3]{x}$ but is **(a)** horizontally compressed where $a = 4$, **(b)** horizontally stretched where $a = 1/4$.

Definition. When the function $y = f(x)$ is multiplied by -1 , the graph of the new function $y = -f(x)$ is the *reflection about the x -axis* of the graph of the function $y = f(x)$.

Definition. When the graph of the function $y = f(x)$ is known, the graph of the new function $y = f(-x)$ is the *reflection about the y -axis* of the graph of the function $y = f(x)$.

Example. Find a formula for the function whose graph is the graph of $y = \sqrt[3]{x}$ but is **(a)** reflected about the x -axis, **(b)** reflected about the y -axis.

Examples. Page 103 numbers 32, 34, and Page 104 number 36.

Examples. Page 104 numbers 46, 60, 66, and Page 105 number 76.

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