Chapter 3. Linear and Quadratic Functions

Section 3.1. Properties of Linear Functions and Linear Models

Note. In this section we define and graph linear functions, determine whether a linear function is increasing/decreasing/constant, and build linear models from verbal descriptions.

Definition. A *linear function* is a function of the form $f(x) = mx + b$. Functions that are not linear are *nonlinear*.

Note. The graph of a linear function $f(x) = mx + b$ is a line with slope m and y-intercept b. The domain is all real numbers $(-\infty, \infty) = \mathbb{R}$. But a **warning**: You are likely to encounter "linear functions" in your future math classes (such as Linear Algebra, MATH 2010) and the definition we use here is different from the definition used elsewhere (in these other settings, a linear function would be required to be of the form $f(x) = mx$).

Example. Page 126 number 18(a)(b).

Theorem 3.1.A. Average Rate of Change of a Linear Function. Linear functions have constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is $\Delta y/\Delta x = m$.

Examples. Page 126 numbers 18(c) and 22.

Note. We know from the definition of slope in Section 1.3, Lines, that a positive slope line has a graph going "uphill" and a negative slope line has a slope going "downhill" (as read from left to right). This allows us to relate slopes to increasing/decreasing for linear functions as follows.

Theorem 3.1.B. Increasing, Decreasing, and Constant Linear Functions. A linear function $f(x) = mx + b$ is increasing over its domain if its slope m is positive. It is decreasing over its domain if its slope m is negative. It is constant over its domain if its slope m is zero.

Example. Page 126 numbers 18(d).

Example. Page 127 number 30.

Note. We now illustrate the building of linear models from verbal descriptions with examples (also notice the Supply and Demand example, Pages 125 and 126 Example 5).

Examples. Page 128 number 38 and Page 129 number 50.