

Section 4.3. The Graph of a Rational Function

Note. In this section we analyze the graph of a rational function and solve applied problems involving rational functions.

Note. The text book lists the following seven steps in analyzing the graph of a rational function R :

Step 1. Factor the numerator and denominator of R . Find the domain of the rational function.

Step 2. Write R in lowest terms.

Step 3. Locate the intercepts, if any, of the graph. The x -intercepts, if any, of $R(x) = p(x)/q(x)$ in lowest terms, are numbers in the domain that satisfy the equation $p(x) = 0$. The y -intercept, if there is one, is $R(0)$. Use multiplicity to determine the behavior of the graph of R at each x -intercept.

Step 4. Find the vertical asymptotes. These occur where the denominator of R (in lowest terms) is 0. Determine the behavior of the graph of R on either side of each vertical asymptote.

Step 5. Locate the horizontal or oblique asymptotes, if any, using the procedure given earlier. Graph the asymptote using a dashed line. Determine points, if any, at which the graph of R intersects these asymptotes.

Step 6. Use the zeros of the numerator and denominator of R to divide the x -axis into intervals. Determine where the graph of R is above or below the x -axis

by choosing a number in each interval and evaluating R there. Plot the points found.

Step 7. Use the results obtained in Steps 1 through 6 to graph R .

Note. Again, the examples in the text book are very good and well-explained. Examples 1 through 5 illustrate the use of the 7 steps.

Examples. Page 211 numbers 8, 30, and 34 Page 212 number 52, and Page 213 number 60.

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