Section 4.4. Polynomial and Rational Inequalities

Note. In this section we solve inequalities involving polynomials and rational functions.

Note. If we have a graph of a function f, then we can easily solve inequalities of the forms $f(x) < 0$ $f(x) \leq 0$, $f(x) > 0$, and $f(x) \geq 0$.

Example. Page 218 number 8.

Note. We can solve polynomial inequalities algebraically using the following steps.

Step 1. Write the inequality so that a polynomial expression f is on the left side and zero is on the right side.

Step 2. Determine the real zeros (x-intercepts of the graph) of f.

Step 3. Use the zeros found in Step 2 to divide the real number line into intervals.

Step 4. Select a number in each interval, denoted c, evaluate f at the number, and determine whether $f(c)$ is positive or negative. If $f(c)$ is positive, all values of f in the interval are positive. If $f(c)$ is negative, all values of f in the interval are negative.

Example. Page 219 number 26.

Note. We can solve polynomial *and* rational inequalities algebraically using the following steps.

Step 1. Write the inequality so that a rational expression f is on the left side and zero is on the right side.

Step 2. Determine the real numbers at which the expression f equals zero and, if the expression is rational, the real numbers at which the expression f is undefined.

Step 3. Use the numbers found in Step 2 to divide the real number line into intervals.

Step 4. Select a number in each interval, denoted c, evaluate f at the number, and determine whether $f(c)$ is positive or negative. If $f(c)$ is positive, all values of f in the interval are positive. If $f(c)$ is negative, all values of f in the interval are negative.

Examples. Page 219 number 48 and 64, and Page 220 number 76.

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