Section 5.5. Properties of Logarithms

Note. In this section we work with properties of logarithms, in particular expressions involving sums and differences of logarithms, and numerically approximate logarithms involving bases other than 10 or e.

Theorem 5.5.A. Properties of Logarithms. Let M, N, and a be positive real numbers where $a \neq 1$, and r any real number.

1. $a^{\log_a M} = M$.

2. $\log_a a^r = r$.

3. The Log of a Product Equals the Sum of the Logs:

$$\log_a(MN) = \log_a M + \log_a N.$$

4. The Log of a Quotient Equals the Difference of the Logs:

$$\log_a(M/N) = \log_a M - \log_a N.$$

5. The Log of a Power Equals the Product of the Power and the Log:

$$\log_a M^r = r \log_a M \text{ and } a^r = e^{r \ln a}.$$

Examples. Page 305 numbers 20, 36, 48, 56, 62, and 70.

Note. The next result follows from the fact that logarithm functions are one to one.

Theorem 5.5.B. Properties of Logarithms. Let M, N, and a be positive real numbers, $a \neq 1$. If $M = N$ then $\log_a M = \log_a N$. If $\log_a M = \log_a N$, then $M = N$.

Note. A standard scientific calculator has keys for the common logarithm and the natural logarithm. If we want to numerically approximate a logarithm function to another base, then we need the following result.

Theorem 5.5.C. Change-of-Base Formula. If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then $\log_a M = \dfrac{\log_b M}{\log_b a}$.

Example. Page 306 number 72, 98, 102.

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