A.10. nth Roots; Rational Exponents

Note. In this appendix, we work with nth roots, simplify radicals, rationalize denominators, solve radical equations, and simplify expressions with rational exponents.

Definition. The (principal) *nth root of real number a*, where $n \geq 2$ is an integer, is defined as

$$\sqrt[n]{a} = b \text{ means } a = b^n$$

where $a \geq 0$ and $b \geq 0$ when n is even, and a and b are any real numbers if n is odd.

Note A.10.A. Some properties of roots include:

(1) $\sqrt[n]{a^n} = a$ if $n \geq 3$ is odd.

(2) $\sqrt[n]{a^n} = |a|$ if $n \geq 2$ is even.

(3) $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ if $a \geq 0$ and $b \geq 0$.

(4) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ if $a \geq 0$ and $b \geq 0$.

(5) $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$.

Note. To "simplify" an expression involving radicals means to remove from radicals any perfect roots that occur as factors (perfect roots are expressions involving radicals which can be simplified as rational numbers: $\sqrt[3]{\dfrac{1}{8}} = \dfrac{1}{2}$).

Examples. Page A88 numbers 12, 14, 22, 30, 36, and 38.

Note. When radicals occur in quotients, it is customary to rewrite the quotient so that the denominator contains no radicals. This process is called "rationalizing the denominator." We often take advantage of the identity $(a+b)(a-b) = a^2 - b^2$.

Examples. Page A88 numbers 52 and 58.

Definition. We now give meaning to rational exponents using radicals. If a is a real number and $n \geq 2$ is an integer, define $a^{1/n} = \sqrt[n]{a}$, provided $\sqrt[n]{a}$ exists. If a is a real number and m and n are integers containing no common factors, with $n \geq 2$, then define $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$, provided $\sqrt[n]{a}$ exists.

Examples. Page A88 numbers 66, 80, 88, and page A89 number 112.

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