# A.5. Rational Expressions

**Note.** In this appendix, we reduce, multiply, divide, add, subtract, and simplify rational expressions.

**Definition.** A *rational expression* is a quotient of polynomials. The polynomial in the role of the dividend is the *numerator* (i.e., it's on top) and the polynomial in the role of the divisor is the *denominator* (i.e., it's on the bottom). When the numerator and denominator of a rational expression have no common factors (except for 1 and $-1$), the rational expression is *reduced to lowest terms* or *simplified*.

**Note.** When simplifying rational expressions, we have to be careful about domains of the expressions. This is all based on the fact that we cannot divide by 0 (ever, ever, **ever**, **ever!!!**).

**Note A.5.A.** Some properties of rational expressions include:

**1.** $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ if $b \neq 0 \neq d$.

**2.** $\frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ if $b \neq 0 \neq c \neq 0 \neq d$.

**3.** $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$ if $b \neq 0$.

**4.** $\frac{a}{b} \pm \frac{c}{d} = \frac{a \times d}{b \times d} \pm \frac{b \times c}{b \times d} = \frac{ad \pm bc}{bd}$ if $b \neq 0 \neq d$.

**Example.** Page A41 number 8, 16, and 19.

**Note.** When adding rational numbers ("fractions"), we must get a common denominator and do so by finding the least common multiple (LCM) of the two denominators.

**Note.** The "LCM Method" for adding or subtracting rational expressions requires four steps:

**1.** Factor completely the polynomials in the denominator of each rational expression.

**2.** The LCM of the denominator is the product of each of these factors raised to a power equal to the greatest power of the factor in the denominator of the rational expression.

**3.** Rewrite each rational expression using the LCM as the denominator.

**4.** Add the numerators of the rewritten rational expressions which now have the LCM as the common denominator.

**Examples.** Page A42 numbers 28 and 32.

**Definition.** When a quotient has sums or differences of rational expressions in the numerator and/or denominator, the quotient is called a *mixed quotient* (or *compound fraction*). To *simplify* a mixed quotient means to rewrite it as a rational expression reduced to lowest terms.

**Note.** To simplify a mixed quotient:

**1.** Treat the numerator and denominator of the mixed quotient separately and simplify each. If the denominator of the mixed expression is itself a rational expression, then invert the denominator and multiply, OR

**2.** Find the LCM of the denominator of all rational expressions that appear in the mixed quotient. Multiply the numerator and denominator of the mixed quotient by the LCM and simplify the resulting rational expression.

**Examples.** Page A42 numbers 34 (method 2) and 36 (method 1).

*Last Updated: 8/15/2021*