A.9. Interval Notation; Solving Inequalities

Note. In this appendix, we introduce and use interval notation, give properties of inequalities, solve inequalities, and solve inequalities involving absolute value.

Definition. An *inequality in one variable* is a statement involving two expressions, at least one containing the variable, separated by one of the inequality symbols: $<$ (less than), $\leq$ (less than or equal to), $>$ (greater than), or $\geq$ (greater than or equal to). To *solve an inequality* is to find all values of the variable for which the statement is true. These values form the *solution* of the inequality.

Note. To solve inequalities, we need to introduce "interval notation."

Definition. Let a and b be real numbers with $a < b$. Then

A *closed interval*, denoted by $[a, b]$, consists of all real numbers x for which $a \leq x \leq b$.

An *open interval*, denoted by (a, b), consists of all real numbers x for which $a < x < b$.

The *half-open*, or *half-closed*, *intervals* are $(a, b]$ consisting of all real numbers x for which $a < x \leq b$, and $[a, b)$ consisting of all real numbers x for which $a \leq x < b$.

In each of these definitions, a is the *left endpoint* and b is the *right endpoint* of the interval.

Note. We will use the symbols ∞ ("infinity") and $-\infty$ ("negative infinity") to indicate unboundedness in intervals. These are not numbers! If you take calculus, you will use these symbols extensively and, even then, they are not numbers! We have five kinds of unbounded intervals:

$[a, \infty)$ consists of all real numbers x for which $x \geq a$ (i.e., $a \leq x < \infty$).

(a, ∞) consists of all real numbers x for which $x > a$ (i.e., $a < x < \infty$).

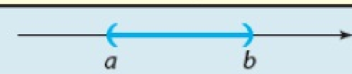
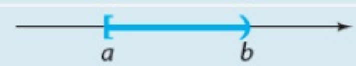
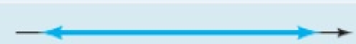
$(-\infty, a]$ consists of all real numbers x for which $x \leq a$ (i.e., $-\infty < x \leq a$).

$(-\infty, a)$ consists of all real numbers x for which $x < a$ (i.e., $-\infty < x < a$).

$(-\infty, \infty)$ consists of all real numbers x (i.e., $-\infty < x < \infty$).

Notice that neither ∞ nor $-\infty$ are endpoints. The first four intervals have only one endpoint and the last interval has no endpoints.

Note. We can also draw pictures of intervals on the number line. From page A73 Table 2 we have:

Interval	Inequality	Graph
The open interval (a, b)	$a < x < b$	
The closed interval $[a, b]$	$a \leq x \leq b$	
The half-open interval $[a, b)$	$a \leq x < b$	
The half-open interval $(a, b]$	$a < x \leq b$	
The interval $[a, \infty)$	$x \geq a$	
The interval (a, ∞)	$x > a$	
The interval $(-\infty, a]$	$x \leq a$	
The interval $(-\infty, a)$	$x < a$	
The interval $(-\infty, \infty)$	All real numbers	

Examples. Page A79 numbers 18, 30, 32, 34, 38, 42.

Note. Throughout this section we deal only with real numbers (there is no "ordering" of the complex numbers and the ideas of greater than/less than make no sense except in the setting of real numbers). The following properties hold for all real numbers:

Nonnegative Property: $a^2 \geq 0$.

Addition Property of Inequalities:

If $a < b$ then $a + c < b + c$.

If $a > b$ then $a + c > b + c$.

Multiplication Properties for Inequalities:

If $a < b$ and if $c > 0$ then $ac < bc$.

If $a < b$ and if $c < 0$ then $ac > bc$.

If $a > b$ and if $c > 0$ then $ac > bc$.

If $a > b$ and if $c < 0$ then $ac < bc$.

Reciprocal Property for Inequalities:

If $a > 0$ then $\frac{1}{a} > 0$.

If $a < 0$ then $\frac{1}{a} < 0$.

These are the properties we use to solve inequalities in one variable. Follow these rules and **DON'T MAKE UP YOUR OWN RULES** and you'll do fine!

Examples. Page A79 numbers 60, 68, 72, Page A80 numbers 86, and 92.

Note. In dealing with inequalities involving absolute value, we use the following.

Theorem A.9.A. If a is any positive number, then:

$|u| < a$ is equivalent to $-a < u < a$.

$|u| \leq a$ is equivalent to $-a \leq u \leq a$.

$|u| > a$ is equivalent to $u < -a$ or $u > a$.

$|u| \geq a$ is equivalent to $u \leq -a$ or $u \geq a$.

Examples. Page A80 numbers 98 and 100.

Example. Page A81 number 123.

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