Chapter 2. Limits and Continuity

2.1. Rates of Change and Tangents to Curves

Definition. The *average rate of change* of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}$$

where $h = x_2 - x_1$.

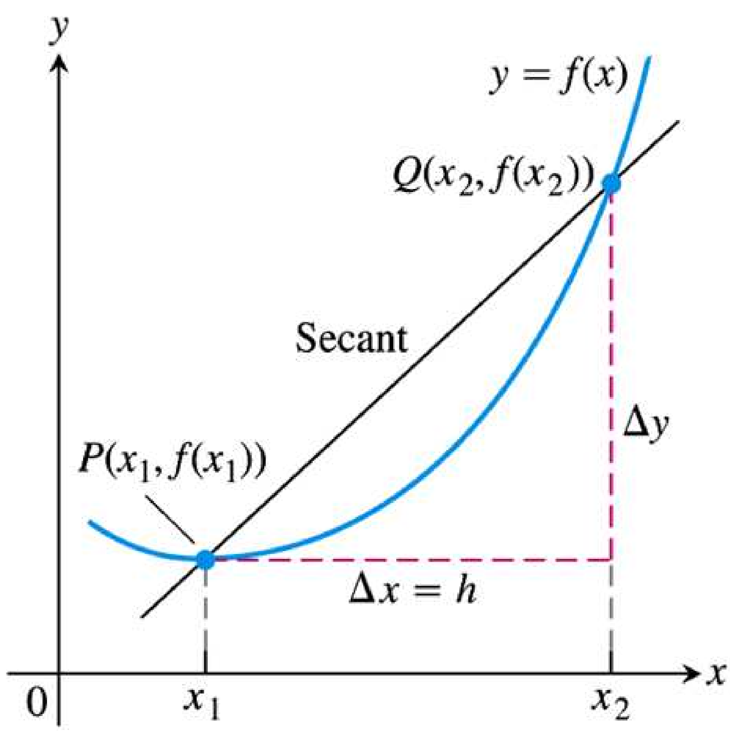

Figure 2.1, page 60

Example. Page 63 number 4a.

Note. We now informally define the *slope of a curve* at a point P on the curve. At this stage, the slope of a *line* is defined, so we use this as a starting point. We define the slope of a curve at a point P as the slope of the line tangent to the curve at point P. To find this tangent line, we approximate it by lines secant to the curve which pass through point P and another point on the curve, say point Q. Since we know two points on the secant line, P and Q, we can find the slope of the secant line. If we make point Q *really close to* point P, then the slope of the secant line should be close to the slope of the tangent line. To find the exact slope of the tangent line, requires that we take a *limit*—and limits are the topic of this chapter.

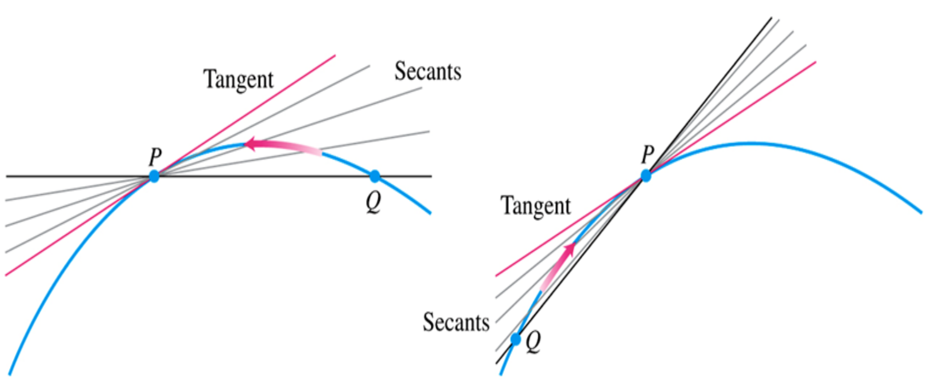

Figure 2.3, page 61

Page 61, Example 3. Find the slope of the parabola $y = x^2$ at the point $P = (2, 4)$. Write an equation for the tangent to the parabola at this point.

Hint: Choose a second point $Q = (2 + h, (2 + h)^2)$ on the curve (where $h \neq 0$) and compute the slope of the secant line PQ. *Guess* what happens to the slope of the secant line when h is close to 0.

Example. Page 64 number 14.