

Chapter 5. Integration

5.5 Indefinite Integrals and the Substitution Method

Note. Suppose we denote a function $f(x)$ as u : $u = f(x)$. Then, for n a rational number, $n \neq -1$, by the Power Rule and Chain Rule for differentiation,

$$\frac{d}{dx} \left[\frac{u^{n+1}}{n+1} \right] = \frac{d}{dx} \left[\frac{(f(x))^{n+1}}{n+1} \right] = (f(x))^{\widehat{n}} [f'(x)].$$

As an indefinite integral, we can write this as

$$\int (f(x))^n f'(x) dx = \frac{(f(x))^{n+1}}{n+1} + C.$$

Since the differential of u is $du = f'(x) dx$, we write

$$\int u^n du = \frac{u^{n+1}}{n+1} + C.$$

Example. Page 343 number 20.

Theorem 5. The Substitution Rule. If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I , then

$$\int f(g(x))g'(x) dx = \int f(u) du.$$

Proof. This can be established by the Chain Rule.

QED

Examples. Page 343 number 32, Page 341 Example 9, Page 376 numbers 50 and 62.