Appendices

A.4. Proofs of Limit Theorems

Note. In this appendix we prove some parts of Theorem 2.1, Limit Rules. We give ε/δ proofs, based on the definition of limit, of the Product Rule for Limits and the Quotient Rule for Limits of Theorem 2.1, plus a proof of the Sandwich Theorem, Theorem 2.4 (all from Section 2.2).

Theorem 2.1(4). Limit Product Rule.

If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then

$$\lim_{x\to c}(f(x)g(x)) = \left(\lim_{x\to c} f(x)\right)\left(\lim_{x\to c} g(x)\right) = LM.$$

Theorem 2.1(5). Limit Quotient Rule.

If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to x} g(x)} = \frac{L}{M},$$

if $\lim_{x\to c} g(x) = M \neq 0$.

Theorem 2.4. Sandwich Theorem.

Suppose that $g(x) \leq f(x) \leq h(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself. Suppose also that

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L.$$

Then $\lim\limits_{x\to c} f(x) = L$.

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