Chapter 3. Derivatives

3.10. Related Rates

Note. In this section we find a relationship between various quantities, each of which changes with time. We then differentiate the relationship implicitly with respect to time, introducing derivatives of the various quantities with respect to time. A derivative with respect to time gives a rate of change, and the implicit differentiation gives a relationship between these rates of change. We start with an example and then give a general strategy for these types of problems.

Example. Exercise 3.10.23 (a sliding ladder).

Note. We largely followed the steps given next in working the sliding ladder problem.

Note. Related Rates Problem Strategy.

We will follow this protocol when solving related rates problems:

Step 1. *Draw a picture and name the variables and constants.* Use t for time. Assume that all variables are differentiable functions of time.

Step 2. *Write down the numerical information* (in terms of the symbols you have chosen).

Step 3. *Write down what you are asked to find* (usually a rate, expressed as a derivative).

Step 4. *Write an equation that relates the variables.* You may have to combine two or more equations to get a single equation that relates the variable whose rate you want to the variables whose rates you know.

Step 5. *Differentiate with respect to t.* Then express the rate you want in terms of the rate and variables whose values you know.

Step 6. *Evaluate.* Use known values to find the unknown rate.

Note. We have no new techniques to introduce in this section, so we just illustrate the strategy with several examples. Useful formula and ideas include:

- The Pythagorean Theorem for right triangles,
- ratios of edges of similar triangles,
- the volume V and surface area S of a sphere of radius r: $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$,
- the volume of a (right circular) cylinder of radius r and height h: $V = \pi r^2 h$,
- the surface area of a cylinder of radius r and height h: $S = 2\pi r h$ (excluding the flat ends), or $S = 2\pi r h + 2\pi r^2$ (including the flat ends),
- the volume of a (right circular) cone of base radius r and height h: $V = \frac{1}{3}\pi r^2 h$.

Examples. Exercise 3.10.28 (a draining conical reservoir), Exercise 3.10.34 (making coffee), Exercise 3.10.40 (a building shadow), Exercise 3.10.44 (ships), and Practice Exercise 3.148 (Motion of a Particle).

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