# **Chapter 3.** Derivatives

## **3.5.** Derivatives of Trigonometric Functions

**Note.** In Section 2.4. One-Sided Limits, we used the Sandwich Theorem (Theorem 2.4) to show that $\lim_{h\to 0} \frac{\sin h}{h} = 1$ (in Theorem 2.7) and $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$ (in Example 2.4.5(a)). We now use these limits to find the derivatives of sine and cosine; then we can find the derivatives of the remaining four trigonometric functions.

**Note.** Recall the summation formula for $\sin x$ which states that for all real numbers $a$ and $b$,

$$\sin(a + b) = \sin a \cos b + \cos a \sin b.$$

**Theorem 3.5.A. Derivative of the Sine Function**

$$\frac{d}{dx}[\sin x] = \cos x$$

**Example.** Exercise 3.5.2.

**Note.** Recall the summation formula for $\cos x$ which states that for all real numbers $a$ and $b$,

$$\cos(a + b) = \cos a \cos b - \sin a \sin b.$$

**Theorem 3.5.B. Derivative of the Cosine Function**

$$\frac{d}{dx}[\cos x] = -\sin x$$

**Example.** Example 3.5.3, Simple Harmonic Motion.

**Examples.** Exercise 3.5.28, Example 3.5.5, and Exercise 3.5.60.

**Theorem 3.5.C.** In summary, we have the following derivatives of the six trigonometric functions:

| $f$ | $f'$ |
|---|---|
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\cot x$ | $-\csc^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\csc x$ | $-\csc x \cot x$ |

**Example.** Exercise 3.5.40 and Exercise 3.5.50.

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