Chapter 4. Applications of Derivatives

4.6. Applied Optimization

Note. In this section we use our knowledge of finding local and absolute extrema of a function to apply them to various applications. We introduce no new techniques, so we largely present examples.

Example. Exercise 4.6.2.

Note. The text book states the following steps which should be used in solving the problems of this section. We followed these steps in the previous example.

Note. Solving Applied Optimization Problems:

1. Read the problem.

2. Draw a picture and label parts that may be important to the problem.

3. Introduce variables (or "unknowns"). Find relationships between the unknowns.

4. Write an equation for the unknown quantity, using the relations from (3) to express the unknown quantity as a function of one variable.

5. Maximize/Minimize the function; test the critical points and endpoints of the domain (if present) of the unknown.

Note. The first step goes without saying! We usually combine steps 2 and 3. The trickiest part is finding relationships between the unknowns and expressing the unknown quantity as a function of a single variable.

Examples. Exercise 4.6.8, Exercise 4.6.12, Exercise 4.6.24, Exercise 4.6.42, Exercise 4.6.52, and Chapter 4 Practice Exercise 112.

Note. Let x be the number of items produced and sold by a business. Let $r(x)$ be the revenue that results from selling the items and let $c(x)$ be the cost of producing the items; these are the *cost function* and *revenue functions*, respectively. The *profit function* is $p(x) = r(x) - c(x)$. Economists call $r'(x)$, $c'(x)$, and $p'(x)$ the *marginal revenue*, *marginal cost*, and *marginal profit* functions, respectively. The *average cost function* is $c(x)/x$. If r and c are differentiable and x can be any value, then profit will be maximized when $p'(x) = r'(x) - c'(x) = 0$; that is, at a production level yielding maximum profit, marginal revenue equals marginal cost (i.e., $r'(x) = c'(x)$). See Figure 4.45.

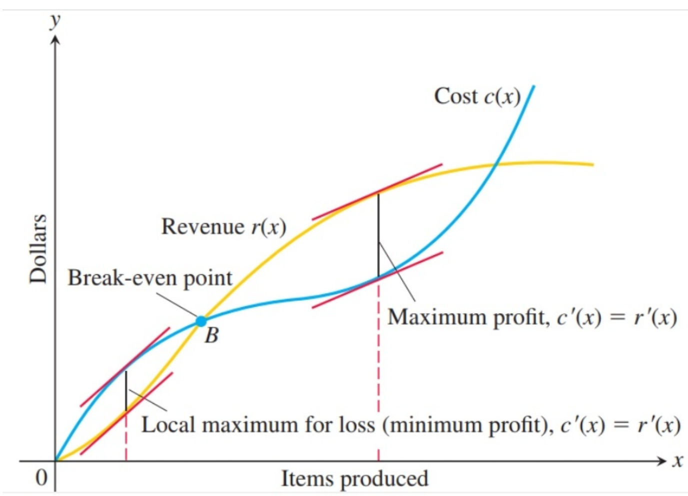

Figure 4.45

Example. Exercise 4.6.62.

Note. There are several additional examples in this section of the book which further illustrate the ideas of applied optimization.

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