

Calculus 1, Chapter 3 “Differentiation”

Study Guide

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The following is a *brief* list of topics covered in Chapter 2 of *Thomas' Calculus*.

3.1 Tangents and the Derivative at a Point. Slope of a Curve, tangent line, difference quotient, derivative at a point, instantaneous velocity, vertical tangent line.

3.2 The Derivative as a Function. Definition of Derivative, interpretations of $y' = f'$ as rates of change, differentiable on an open interval, differentiable on a closed interval, right/left-hand derivatives, points of non-differentiability (corner, cusp, vertical tangent, discontinuity, wild oscillation), Differentiability Implies Continuity (Theorem 3.1), the converse of Theorem 3.1 does not hold, Alternative Formula Derivative (Exercise 3.2.24).

3.3 Differentiation Rules. Derivative of a Constant Function (Theorem 3.3.A), Derivative Power Rule for Positive Integers (Theorem 3.3.B), Derivative Power Rule General Version (Theorem 3.3.C), Derivative Constant Multiple Rule (Theorem 3.3.D), Derivative Sum Rule (Theorem 3.3.E), linear operator, derivative of a polynomial (Exercise 3.3.73), Derivative of the Natural Exponential Function (Theorem 3.3.F), Derivative Product Rule (Theorem 3.3.G), Derivative Quotient Rule (Theorem 3.3.H), my square bracket notation, extended product rule (Exercise 3.3.77), higher order derivatives.

3.4 The Derivative as a Rate of Change. Instantaneous rate of change, displacement, average velocity, instantaneous velocity, speed, rectilinear motion, acceleration, jerk, “marginal.”

3.5 Derivatives of Trigonometric Functions. Derivative of sine (Theorem 3.5.A), derivative of cosine (Theorem 3.5.B), simple harmonic motion (Example 3.5.3), derivatives of the rest of the trig functions (Theorem 3.5.C).

3.6 The Chain Rule. Interpretation of the Chain Rule as a system of gears, compositions of functions, The Chain Rule (Theorem 3.2), using “little arrows” to represent the use of the Chain Rule, the use of all the rules of differentiation in a single problem (Example 3.6.A).

3.7 Implicit Differentiation. Definition of “function f is implicit to an equation $F(x, y) = 0$ ”, implicit differentiation, finding slopes of curves determined by a formula, normal line, higher order derivatives found implicitly.

3.8 Derivatives of Inverse Functions and Logarithms. Inverse function, The Derivative Rule for Inverses (Theorem 3.3), derivatives of logarithm functions (Theorem 3.8.A), logarithmic differentiation, derivatives of exponentials base a (Theorem 3.8.B), differentiation of logarithms base a (Theorem 3.8.C), General Power Rule for Derivatives (Theorem 3.3.C/3.8.D), The Number e as a Limit (Theorem 3.4).

3.9 Inverse Trigonometric Functions. Definitions and graphs of inverse trig functions, identities involving inverse trig functions, derivative of $\sin^{-1} u$ (Theorem 3.9.A), derivative of $\tan^{-1} u$ (Theorem 3.9.B), derivative $\sec^{-1} u$ (Theorem 3.9.A), derivatives of the other inverse trigonometric functions.

3.10 Related Rates. The 6 steps involved in the Related Rates Problem Strategy, examples.

3.11 Linearization and Differentials. Linearization, standard approximation, center of an approximation, differentials dx and dy , differentials as estimate of change, absolute/relative/percentage change, Lemma 3.11.A, proof of The Chain Rule (Theorem 3.2).