Chapter 3. Derivatives

3.5. Derivatives of Trigonometric Functions

Note. In Section 2.4. One-Sided Limits, we used the Sandwich Theorem (Theorem 2.4) to show that \(\lim_{h \to 0} \frac{\sin h}{h} = 1\) (in Theorem 2.7) and \(\lim_{h \to 0} \frac{\cos h - 1}{h} = 0\) (in Example 2.4.5(a)). We now use these limits to find the derivatives of sine and cosine; then we can find the derivatives of the remaining four trigonometric functions.

Note. Recall the summation formula for \(\sin x\) which states that for all real numbers \(a\) and \(b\),

\[
\sin(a + b) = \sin a \cos b + \cos a \sin b.
\]

**Theorem 3.5.A.** Derivative of the Sine Function

\[
\frac{d}{dx} [\sin x] = \cos x
\]

**Example.** Exercise 3.5.2.

Note. Recall the summation formula for \(\cos x\) which states that for all real numbers \(a\) and \(b\),

\[
\cos(a + b) = \cos a \cos b - \sin a \sin b.
\]
Theorem 3.5.B. Derivative of the Cosine Function

\[ \frac{d}{dx} [\cos x] = -\sin x \]

Example. Example 3.5.3, Simple Harmonic Motion.

Examples. Exercise 3.5.28, Example 3.5.5, and Exercise 3.5.60.

Theorem 3.5.C. In summary, we have the following derivatives of the six trigonometric functions:

<table>
<thead>
<tr>
<th>f</th>
<th>f'</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin x</td>
<td>cos x</td>
</tr>
<tr>
<td>cos x</td>
<td>- sin x</td>
</tr>
<tr>
<td>tan x</td>
<td>sec^2 x</td>
</tr>
<tr>
<td>cot x</td>
<td>- csc^2 x</td>
</tr>
<tr>
<td>sec x</td>
<td>sec x tan x</td>
</tr>
<tr>
<td>csc x</td>
<td>- csc x cot x</td>
</tr>
</tbody>
</table>

Example. Exercise 3.5.40 and Exercise 3.5.50.

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