Calculus 1 Test 1 — Fall 2012

NAME KEY STUDENT NUMBER ____________

SHOW ALL WORK!!! Partial credit will only be given for answers which are *partially correct*! Be clear and convince me that you understand what is going on. Use equal signs when things are equal (and don't use them when things are not equal). Be sure to include all necessary symbols (such as limits). Your grade is based on the work you show and the arguments you make. If applicable, put your answer in the box provided. Each numbered problem is worth 12 points. **No calculators.**

$\boxed{\text{p 11 \#3}}$

1. Find the domain of $F(x) = \sqrt{5x+10}$.

Well, we need $5x + 10 \geq 0$, or $x \geq -2$. So the domain is $x \in [-2, \infty)$

$$\boxed{[-2, \infty)}$$

TYPO: cos

$\boxed{\text{p 63 \#4}}$

2. Find the average rate of change of $g(t) = 2 + \cancel{\cot}\, t$ on the interval $[0, \pi]$.

Well, average rate of change is

$$\frac{\Delta g}{\Delta t} = \frac{g(t_2) - g(t_1)}{t_2 - t_1} = \frac{(2 + \cos(\pi)) - (2 + \cos(0))}{\pi - 0}$$

$$= \frac{(2 + (-1)) - (2 + (1))}{\pi - 0} = \frac{1 - 3}{\pi} = \frac{-2}{\pi}$$

$$\boxed{\frac{-2}{\pi}}$$

3. Evaluate $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$, if it exists.

p74 #36

Well, $\lim_{x\to 1}\left(\frac{x-1}{\sqrt{x+3}-2}\right) = \lim_{x\to 1}\left(\frac{x-1}{\sqrt{x+3}-2}\right)\left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right)$

$= \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-(4)} \stackrel{E}{=} \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x-1)} \stackrel{C}{=} \lim_{x\to 1}(\sqrt{x+3}+2)$

$\stackrel{S}{=} \sqrt{(1)+3}+2 = \sqrt{4}+2 = 2+2 = 4$

$\boxed{4}$

4. State the **precise** (or **formal**) definition of $\lim_{x\to x_0} f(x) = L$ (the definition with the ϵ and δ in it).

p.77 Let $f(x)$ be defined on an open interval about x_0 except possibly x_0 itself. We say that $f(x)$ approaches limit L as x approaches x_0 and write $\lim_{x\to x_0} f(x) = L$ if, for every number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x,

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$

5. Consider $f(x) = \sqrt{4-x^2}$. Evaluate $\lim_{x\to 2} f(x)$, if it exists. Give a clear explanation of your answer!

p 86 Eg 1

Well, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}\sqrt{4-x^2} = \lim_{x\to 2^-}\sqrt{\underset{+}{(2-x)}\underset{+}{(2+x)}} = \sqrt{0} = 0$

and

Notice the WARNING SIGN!

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}\sqrt{4-x^2} = \lim_{x\to 2^+}\sqrt{\underset{-}{(2-x)}\underset{+}{(2+x)}}$ does not exist (square roots of negatives).

So, the two sided limit $\lim_{x\to 2} f(x)$ does not exist!

$\boxed{\text{does not exist}}$

6. Evaluate $\lim_{x\to 0} \frac{x + x\cos x}{\sin x \cos x}$, if it exists.

p92 #29

Well, $\lim_{x\to 0} \frac{x + x\cos(x)}{\sin(x)\cos(x)} = \lim_{x\to 0} \frac{x(1+\cos x)}{\sin(x)\cos(x)}$

$= \lim_{x\to 0}\left(\frac{x}{\sin x}\right) \lim_{x\to 0}\left(\frac{1+\cos(x)}{\cos x}\right) = (1)\left(\frac{1+\cos(0)}{\cos(0)}\right)$

$= (1)\left(\frac{1+(1)}{(1)}\right) = 2.$

$\boxed{2.}$

7. Consider $f(x) = \begin{cases} x^2 - 1 & \text{if } x \in [-1, 0) \\ 2x & \text{if } x \in (0,1) \\ 1 & \text{if } x = 1 \\ -2x+4 & \text{if } x \in (1,2) \\ 0 & \text{if } x \in (2,3) \end{cases}$. Is $f(x)$ continuous at $x = 1$? Explain.

p101 #6

Well, ① $f(1) = 1$ exists ✓

② $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(2x) = 2(1) = 2$ and

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(-2x+4) = -2(1)+4 = 2.$

So $\lim_{x\to 1} f(x) = 2$ exists ✓

$\boxed{\text{NO}}$

③ BUT $1 = f(1) \neq \lim_{x\to 1} f(x) = 2$, and

so f is NOT continuous at $x = 1$.

8. Consider $f(x) = \dfrac{x+3}{x+2}$. Find the horizontal and vertical asymptotes of f and graph $y = f(x)$ in such a way as to reflect the asymptotes.

$\boxed{\text{p115 \#67}}$

For H.A.: $\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \left(\dfrac{x+3}{x+2}\right) = \lim\limits_{x \to \pm\infty} \left(\dfrac{x+3}{x+2}\right)\left(\dfrac{1/x}{1/x}\right)$

$$= \lim_{x \to \pm\infty} \left(\frac{1+\frac{3}{x}}{1+\frac{2}{x}}\right) = \frac{\lim\limits_{x \to \pm\infty}(1) + 3\lim\limits_{x \to \pm\infty}\left(\frac{1}{x}\right)}{\lim\limits_{x \to \pm\infty}(1) + 2\lim\limits_{x \to \pm\infty}\left(\frac{1}{x}\right)} = \frac{1+3(0)}{1+2(0)} = 1.$$

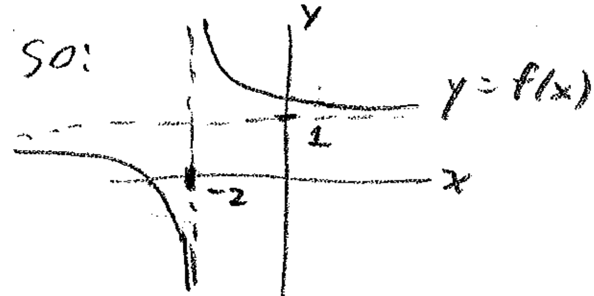
So, $y = 1$ is H.A.

For V.A. consider

$$\lim_{x \to -2^-} \left(\frac{x+3}{x+2}\right) \overset{\frac{(+)}{(-)}}{=\!=} -\infty$$

$$\lim_{x \to -2^+} \left(\frac{x+3}{x+2}\right) \overset{\frac{(+)}{(+)}}{=\!=} +\infty$$

So:



$\boxed{\begin{array}{ll} \text{H.A.} & y = 1 \\ \text{V.A.} & x = 2 \end{array}}$

$\boxed{\text{p.72}}$ **Bonus.** (a) State the Sandwich Theorem.

Suppose that $g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, except possibly $x = c$ itself. Suppose also that $\lim\limits_{x \to c} g(x) = \lim\limits_{x \to c} h(x) = L$.

Then $\lim\limits_{x \to c} f(x) = L$.

$\boxed{\text{p. 89}}$ (b) Suppose that $1 > \dfrac{\sin\theta}{\theta} > \cos\theta$ for $0 < \theta < \pi/2$. Then find $\lim\limits_{\theta \to 0^+} \dfrac{\sin\theta}{\theta}$.

Well, we have $\cos\theta \le \dfrac{\sin\theta}{\theta} \le 1$, so by the Sandwich Theorem (applied to a one sided limit), since

$\lim\limits_{\theta \to 0^+} (\cos\theta) = \cos(0) = 1 = \lim\limits_{\theta \to 0^+} (1)$, then

$$\lim_{\theta \to 0^+} \left(\frac{\sin\theta}{\theta}\right) = 1$$

$\boxed{1}$