Calculus 1 Test 1 (Online) — Fall 2020

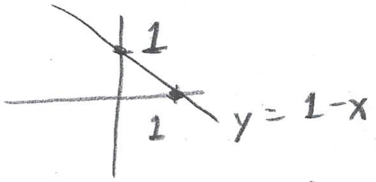
NAME KEY ____________________ STUDENT NUMBER ____________________

SHOW ALL WORK!!! Partial credit will only be given for answers which are *partially correct*! Be clear and convince me that you understand what is going on. Use equal signs when things are equal (and don't use them when things are not equal). Be sure to include all necessary symbols (such as limits). **Write in complete sentences!** When asked to explain, give justifications using the definitions and theorems in the notes and book (quote them by name or number, as we did in class). Your grade is based on the work you show and the arguments you make. If applicable, put your final answer in the box provided. Each numbered problem is worth 20 points.

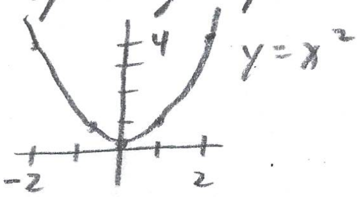
You are not to collaborate with anyone during the test! When you are done, scan your solutions and put them in the Dropbox in D2L. If you have trouble with D2L (which I wouldn't expect), then e-mail me your solutions at `gardnerr@etsu.edu` (but please try to get your solutions submitted in D2L). I prefer PDFs, but if things aren't working then go ahead and just submit photographs of your solutions. The test is due by 11:15.

1. Graph the function $f(x) = \begin{cases} 1-x, & x \in (-\infty, 0) \\ 1, & x = 0 \\ x^2 & x \in (0, \infty). \end{cases}$ Briefly explain your answer.

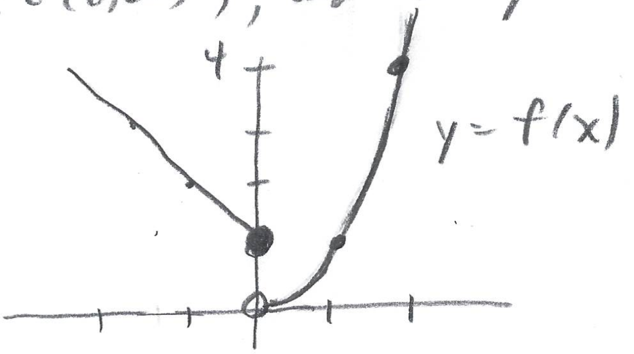
The graph of $y = 1 - x$ is a line of slope $m = -1$ with y-intercept $x = 1$, and so is



The graph of $y = x^2$ is an opening upward parabola with vertex $(0,0)$, and so is



We graph the line for $x < 0$ (i.e., $x \in (-\infty, 0)$), the parabola for $x > 0$ (i.e., $x \in (0, \infty)$), and the point $(0, 1)$, so:



2. The population of Johnson City was 67,000 in 2018. The population increases according to the model for exponential growth at a rate of 1.6% per year. **(a)** What will the population be in 2030? **(b)** When will the population reach 100,000? Give exact values (though you can also give numerical approximations if you like).

The model is $y = y_0 e^{kt}$ where y_0 is the initial population size, t is the time in years, and k is the growth constant. We measure time from 2018 (so we have $t=0$ in year 2018). Here, $y_0 = 67{,}000$ and $k = 1.6\% = 0.016$.

(a) In 2030, $t = 2030 - 2018 = 12$, so the population is $y = 67{,}000\, e^{(0.016)(12)} \approx 81{,}182$

(b) We set $y = 100{,}000$ and solve for t:

$100{,}000 = 67{,}000\, e^{(0.016)t}$ or $\frac{100}{67} = e^{0.016t}$

or $\ln\left(\frac{100}{67}\right) = \ln\left(e^{0.016t}\right) = 0.016t$ or

$t = \dfrac{\ln\left(\frac{100}{67}\right)}{0.016} \approx 25.03$. So the population reaches this level in the year $2018 + \dfrac{\ln\left(\frac{100}{67}\right)}{0.016} \approx 2043$

(a) $e^{(0.016)(12)} \approx 81{,}182$

(b) $2018 + \dfrac{\ln(100/67)}{0.016}$ ≈ 2043

3. Evaluate $\lim_{x \to 5} \dfrac{x^2 - 3x - 10}{x^2 - 25}$, if it exists. If it does not exist then explain why. **Justify your steps!** HINT: Use the Factor/Cancel/Substitute technique.

We have $\displaystyle \lim_{x \to 5} \frac{x^2 - 3x - 10}{x^2 - 25} = \lim_{x \to 5} \frac{(x-5)(x+2)}{(x-5)(x+5)}$ (FACTOR)

$\displaystyle = \lim_{x \to 5} \frac{x+2}{x+5}$ by Dr. Bob's Limit Theorem (CANCEL)

$\displaystyle = \frac{(5)+2}{(5)+5}$ by Theorem 2.3, Limits of Rational Functions (SUBSTITUTE)

$\displaystyle = \frac{7}{10}.$

$$\boxed{\frac{7}{10}}$$

4. Consider $f(x) = \sqrt{13-x}$, $L = 3$, $c = 4$, and $\varepsilon = 1$. **(a)** Find an open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. **(b)** Then give a value for $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

(a) We want $|f(x)-L|<\varepsilon$ or $|\sqrt{13-x}-3|<1$ or $-1<\sqrt{13-x}-3<1$ or $2<\sqrt{13-x}<4$ or $(2)^2<(\sqrt{13-x})^2<(4)^2$ or $4<13-x<16$ or $-9<-x<3$ or $-3<x<9$.

Since each step is reversible, we take $x \in (-3, 9)$.

(b) We consider the distance from $c=4$ to the endpoints. The distance from $c=4$ to -3 is $4-(-3)=7$. The distance from $c=4$ to 9 is $9-4=5$. We choose as the smaller of 5 and 7, so we take $\delta=5$.

Then $0<|x-c|<\delta$ implies $|x-4|<5$ or $-5<x-4<5$ or $-1<x<9$ or $x \in (-1,9) \subset (-3,9)$ so that the desired inequality $|f(x)-L|<\varepsilon$, or $|\sqrt{13-x}-3|<1$, holds.

(a) $(-3, 9)$

(b) $\delta = 5$

5. Evaluate the limits. Justify your computations.

(a) Evaluate $\lim_{x\to 4+} (x-7)\dfrac{|4-x|}{4-x}$.

(b) Evaluate $\lim_{x\to 0} \dfrac{(2\sin x)(1-\cos x)}{x^2}$.

(a) For $x \to 4^+$ we have $x > 4$ so that $4-x < 0$ and hence $|4-x| = -(4-x) = x-4$. So

$$\lim_{x\to 4^+} (x-7)\frac{|4-x|}{4-x} = \lim_{x\to 4^+} (x-7)\frac{-(4-x)}{4-x}$$

$$= \lim_{x\to 4^+} -(x-7)$$ by Dr. Bob's Limit Theorem

$$= -((4)-7)$$ by Theorem 2.2, Limits of Polynomials

$$= 3.$$

(b) We have $$\lim_{x\to 0}\frac{(2\sin x)(1-\cos x)}{x^2} = \lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)\left(\frac{1-\cos x}{x}\right)$$

$$= \lim_{x\to 0} -2\left(\frac{\sin x}{x}\right)\left(\frac{\cos x-1}{x}\right) = -2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\lim_{x\to 0}\left(\frac{\cos x-1}{x}\right)$$

by Theorem 2.1 (3 and 4), Constant Multiple and Product Ru

$$= -2(1)(0)$$ by Theorem 2.7 and Example 2.4.5(a)

$$= 0.$$

(a)	3
(b)	0