# Honors Calculus 1 Test 2 — Fall 2011

NAME KEY STUDENT NUMBER

**SHOW ALL WORK!!!** Partial credit will only be given for answers which are *partially correct*! Be clear and convince me that you understand what is going on. Use equal signs when things are equal (and don't use them when things are not equal). Be sure to include all necessary symbols (such as limits). Use the square bracket notation when taking derivatives. Your grade is based on the work you show and the arguments you make. If applicable, put your answer in the box provided. Each numbered problem is worth 12 points. **No calculators.**

**1.** *Use the definition* of "slope of a curve" to find the slope of $y = 2\sqrt{x}$ at the point $(x_0, y_0) = (1, 2)$. (If you like, you may *use the definition* of derivative.)

p125 #7

Well,

$$m = \lim_{h\to 0}\left(\frac{f(x_0+h)-f(x_0)}{h}\right) = \lim_{h\to 0}\left(\frac{2\sqrt{1+h}-2\sqrt{1}}{h}\right)\left(\frac{2\sqrt{1+h}+2}{2\sqrt{1+h}+2}\right)$$

$$= \lim_{h\to 0}\frac{4(1+h)-4}{h(2\sqrt{1+h}+2)} \overset{F}{=} \lim_{h\to 0}\frac{4h}{h(2\sqrt{1+h}+2)} \overset{C}{=} \lim_{h\to 0}\left(\frac{4}{2\sqrt{1+h}+2}\right)$$

$$\overset{S}{=} \lim_{h\to 0}\left(\frac{4}{2\sqrt{1+(0)}+2}\right) = 1$$

$$\boxed{m = 1}$$

**2.** *Use the definition of derivative* to differentiate $f(x) = -1/x$. No credit will be given unless you use the definition!

p134 #50

Well,

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right) = \lim_{h\to 0}\frac{\left(\frac{-1}{x+h}\right)-\left(\frac{-1}{x}\right)}{h} = \lim_{h\to 0}\frac{1}{h}\left(\frac{-1}{x+h}+\frac{1}{x}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{-(x)+(x+h)}{(x+h)x}\right) \overset{F}{=} \lim_{h\to 0}\left(\frac{1}{h}\right)\left(\frac{h}{(x+h)x}\right) \overset{C}{=} \lim_{h\to 0}\frac{1}{(x+h)x}$$

$$\overset{S}{=} \lim_{h\to 0}\frac{1}{(x+(0))x} = \frac{1}{x^2}$$

$$\boxed{f'(x) = \frac{1}{x^2}}$$

3. (a) Differentiate: $f(x) = (x^2+1)\left(x+5+\frac{1}{x}\right)$. Use the square bracket notation and don't simplify.

p 143 #15

$$f'(x) = [2x]\left(x+5+\frac{1}{x}\right) + (x^2+1)\left[1-\frac{1}{x^2}\right]$$

p 143 #20

(b) Differentiate: $f(t) = \frac{t^2-1}{t^2+t-2}$. Use the square bracket notation and don't simplify.

$$f'(t) = \frac{[2t](t^2+t-2) - (t^2-1)[2t+1]}{(t^2+t-2)^2}$$

4. Suppose the position of a body moving on a coordinate line is $s = t^2 - 3t + 2$ where $s$ is in meters and $t$ is in seconds. Find the body's speed and acceleration at $t = 0$.

p 152 #1

Well, velocity $= \frac{ds}{dt} = 2t-3$ and speed $= \left|\frac{ds}{dt}\right| = |2t-3|$.

So when $t=0$, speed $= |2(0)-3| = 3$ m/sec.

Acceleration $= \frac{d^2s}{dt^2} = 2$ m/sec$^2$ (for all $t$)

$\boxed{\begin{array}{l}\text{speed} = 3 \text{ m/sec} \\ \text{accel} = 2 \text{ m/sec}^2\end{array}}$

5. Does the graph of $y = x + \sin x$ have any horizontal tangents in the interval $0 \leq x \leq 2\pi$? If so, where? If not, why not?

p160 #39

Well, the slope of a tangent line is $y' = 1 + \cos(x)$. A horizontal tangent has slope 0, so set $y' = 1 + \cos(x) \equiv 0$. This implies $\cos(x) = -1$ and so $x = \pi$.

$$\boxed{x = \pi}$$

6. Differentiate: $y = \tan^2(\sin^3 t)$. Use the square bracket notation and don't simplify.

p168 #67

$$y' = 2\tan(\sin^3(t))\left[\sec^2(\sin^3(t))\left[3(\sin^2(t))\left[\cos(t)\right]\right]\right]$$

7. Consider the equation $2\sqrt{y} = x - y$. Find $y' = dy/dx$ and $y'' = d^2y/dx^2$.

p174 #25

Well,

$$\frac{d}{dx}\left[2y^{1/2}\right] = \frac{d}{dx}\left[x - y\right] \Rightarrow 2\left[\tfrac{1}{2}y^{-1/2}\right]\left[y'\right] = 1 - \left[y'\right] \Rightarrow$$

$$y'(y^{-1/2} + 1) = 1 \Rightarrow y' = \frac{1}{y^{-1/2} + 1} = (y^{-1/2} + 1)^{-1}.$$

So $$y'' = \frac{d}{dy}[y'] = -(y^{-1/2} + 1)^{-2}\left[-\tfrac{1}{2}y^{-3/2}\left[y'\right]\right]$$

$$= \frac{y'}{2(y^{-1/2} + 1)^2(y^{3/2})}$$

$$= \frac{1}{2(y^{-1/2} + 1)^3(y^{3/2})}$$

$$\boxed{y' = (y^{-1/2} + 1)^{-1}}$$

$$\boxed{y'' = \frac{1}{2(y^{-1/2} + 1)^3(y^{3/2})}}$$

**8.** (a) Differentiate: $y = \ln(\ln(\ln x))$. Use the square bracket notation and don't simplify.

$\boxed{\begin{array}{c} p184 \\ \#30 \end{array}}$

$$y' = \frac{1}{\ln(\ln x)}\left[\frac{1}{\ln x}\left[\frac{1}{x}\right]\right]$$

(b) Differentiate: $y = \ln(\sec\theta + \tan\theta)$. Use the square bracket notation and don't simplify.

$\boxed{\begin{array}{c} p184 \\ \#32 \end{array}}$

$$y' = \frac{1}{\sec\theta + \tan\theta}\left[\sec\theta\tan\theta + \sec^2\theta\right]$$

**Honors Bonus.** Prove: If $f$ has a derivative at $x = c$, then $f$ is continuous at $x = c$.

$\boxed{p131}$ By definition, we need to show that $\lim_{x\to c} f(x) = f(c)$, or equivalently that $\lim_{h\to 0} f(c+h) = f(c)$. Then

$$\lim_{h\to 0} f(c+h) = \lim_{h\to 0}\left(f(c) + \frac{f(c+h)-f(c)}{h}\cdot h\right)$$

$$= \lim_{h\to 0}\left(f(c)\right) + \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}\cdot\lim_{h\to 0}(h)$$

$$= f(c) + f'(c)\cdot 0 = f(c). \quad \blacksquare$$