Calculus 1 Test 2 — Fall 2012

NAME KEY STUDENT NUMBER ____________

SHOW ALL WORK!!! Partial credit will only be given for answers which are *partially correct*! Be clear and convince me that you understand what is going on. Use equal signs when things are equal (and don't use them when things are not equal). Be sure to include all necessary symbols (such as limits). Use the square bracket notation when taking derivatives. Your grade is based on the work you show and the arguments you make. If applicable, put your answer in the box provided. Each numbered problem is worth 12 points. **No calculators.**

1. *Use the definition* of "slope of a curve" (which involves limits) to find the slope of $y = \sqrt{x+1}$ at the point $(x_0, y_0) = (8, 3)$.

p125 #18

$$m = \lim_{h\to 0}\left(\frac{f(x_0+h)-f(x_0)}{h}\right) = \lim_{h\to 0}\left(\frac{\sqrt{(8+h)+1}-\sqrt{(8)+1}}{h}\right) = \lim_{h\to 0}\left(\frac{\sqrt{9+h}-3}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{9+h}-3}{h}\right)\left(\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}\right) = \lim_{h\to 0}\frac{(9+h)-9}{h(\sqrt{9+h}+3)} \overset{F}{=} \lim_{h\to 0}\frac{h}{h(\sqrt{9+h}+3)}$$

$$\overset{C}{=} \lim_{h\to 0}\frac{1}{\sqrt{9+h}+3} \overset{S}{=} \frac{1}{\sqrt{9+(0)}+3} = \frac{1}{6}$$

$$\boxed{\frac{1}{6}}$$

2. *Use the definition of derivative* (which involves limits) to differentiate $f(x) = x + \dfrac{9}{x}$. No credit will be given unless you use the definition!

p132 #13

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right) = \lim_{h\to 0}\frac{\left((x+h)+\frac{9}{(x+h)}\right)-\left(x+\frac{9}{x}\right)}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left(h+\frac{9}{x+h}-\frac{9}{x}\right) = \lim_{h\to 0}\frac{1}{h}\left(h+\frac{9x-9(x+h)}{(x+h)x}\right)$$

$$\overset{F}{=} \lim_{h\to 0}\frac{1}{h}\left(h+\frac{-9h}{(x+h)x}\right) \overset{C}{=} \lim_{h\to 0}\left(1-\frac{9}{(x+h)x}\right)$$

$$\overset{S}{=} 1-\frac{9}{(x+(0))x} = 1-\frac{9}{x^2}$$

$$\boxed{1-\frac{9}{x^2}}$$

x^{-1}

3. (a) Differentiate: $f(x) = (x^2+1)\left(x+5+\frac{1}{x}\right)$. Use the square bracket notation and don't simplify.

p143 #15

$$f'(x) = [2x]\left(x+5+\frac{1}{x}\right) + (x^2+1)[1+0-x^{-2}]$$

$s^{1/2}$

(b) Differentiate: $f(s) = \dfrac{\sqrt{s}-1}{\sqrt{s}+1}$. Use the square bracket notation and don't simplify.

p143 #23

$$f'(s) = \frac{[\frac{1}{2}s^{-1/2}](s^{1/2}+1) - (s^{1/2}-1)[\frac{1}{2}s^{-1/2}]}{(s^{1/2}+1)^2}$$

4. Had Galileo dropped a cannonball from the Tower of Pisa, 179 ft above the ground, the ball's height above the ground t sec into the fall would have been $s = 179 - 16t^2$. What is the cannonball's velocity when it hits the ground? (You may express your answer in terms of radicals and not worry about simplifying.)

p152 #13c

Well, it hits the ground when $s = 179 - 16t^2 = 0 \Rightarrow t^2 = \frac{179}{16}$

$\Rightarrow t = \frac{\sqrt{179}}{4}$. Velocity is $\frac{ds}{dt} = -32t$, so the velocity when it hits the ground is

$$\left.\frac{ds}{dt}\right|_{t=\frac{\sqrt{179}}{4}} = -32\left(\frac{\sqrt{179}}{4}\right) = -8\sqrt{179}$$

$\boxed{-8\sqrt{179} \text{ ft/sec}}$

p169 #39

5. Does the graph of $y = x - \cot x$ have any horizontal tangents in the interval $0 \le x \le 2\pi$? If so, where? If not, why not?

Well, $y' = 1 - (-\csc^2(x))$, so set $y' = 1 + \csc^2(x) \equiv 0$.

Then we need $\csc^2(x) = -1$, but this never happens.

$\boxed{\text{NO!}}$

p168 #62

6. (a) Differentiate: $y = \cos\left(5\sin\left(\frac{t}{3}\right)\right)$. Use the square bracket notation and don't simplify.

$y' = -\sin\left(5\sin\left(\frac{t}{3}\right)\right)\left[5\cos\left(\frac{t}{3}\right)\left[\frac{1}{3}\right]\right]$

p168 #77

(b) Find the second derivative for $y = e^{x^2} + 5x$. Use the square bracket notation and don't simplify.

$y' = e^{x^2}[2x] + 5 = 2xe^{x^2} + 5$, so

$y'' = [2](e^{x^2}) + (2x)\left[e^{x^2}[2x]\right] + 0$

p174 #35

7. Find the slope of the curve determined by the equation $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at the point $(-1, 0)$.

Well, $\frac{d}{dx}[6x^2 + 3xy + 2y^2 + 17y - 6] = \frac{d}{dx}[0]$

$\Rightarrow 12x + [3](y) + (3x)[y'] + 4y[y'] + 17y' = 0$

$\Rightarrow y'(3x + 4y + 17) = -12x - 3y$

$\Rightarrow y' = \dfrac{-12x - 3y}{3x + 4y + 17}$. When $(x, y) = (-1, 0)$,

$y' = \dfrac{-12(-1) - 3(0)}{3(-1) + 4(0) + 17} = \boxed{\dfrac{12}{14} = \dfrac{6}{7}}$

8. (a) Differentiate: $y = \ln(\sec\theta + \tan\theta)$. Use the square bracket notation and don't simplify.

p184 #32

$y' = \dfrac{1}{\sec\theta + \tan\theta}[\sec\theta\tan\theta + \sec^2\theta]$

$(x^2 - 1)^{1/2}$

(b) Differentiate: $y = \tan^{-1}\sqrt{x^2 - 1} + \csc^{-1} x$, $x > 1$. Use the square bracket notation and don't simplify.

p191 #39

$y' = \dfrac{1}{1 + (\sqrt{x^2 - 1})^2}\left[\frac{1}{2}(x^2 - 1)^{-1/2}[2x]\right] + \dfrac{-1}{|x|\sqrt{x^2 - 1}}$

p185 #92

Bonus. Differentiate $y = x^{\sqrt{x}}$.

Well, $\ln(y) = \ln(x^{\sqrt{x}}) = \sqrt{x}\ln x \Rightarrow \frac{d}{dx}[\ln y] = \frac{d}{dx}[\sqrt{x} \cdot \ln x]$

$\Rightarrow \frac{1}{y}[y'] = [\frac{1}{2}x^{-1/2}](\ln x) + (\sqrt{x})[\frac{1}{x}]$

$\Rightarrow y' = y\left(\frac{1}{2}x^{-1/2}\ln(x) + \frac{1}{\sqrt{x}}\right) = \boxed{x^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\ln(x) + \frac{1}{\sqrt{x}}\right)}$