Honors Calculus 1 Test 1 — Fall 2010

NAME KEY STUDENT NUMBER

SHOW ALL WORK!!! Partial credit will only be given for answers which are *partially correct*! Be clear and convince me that you understand what is going on. Communicate with me; write words! Use equal signs when things are equal (and don't use them when things are not equal). Be sure to include all necessary symbols, such as equal signs and limits. If applicable, put your answer in the box provided. Each numbered problem is worth 12 points and the bonus problem is worth 12 points. **Put your calculators away!!! This is a *math* test!!!**

1. (a) Explain the mathematical philosophy of *formalism.*

NOTES p. 1

Formalism is "the assertion of the possibility and desirability of banishing intuitions by showing formal systems to be entirely adequate to the business of mathematics.

(b) Define a *well-formed formula* and what it means for an axiomatic system to be *complete.*

NOTES p. 9

WFF are the claims of an axiomatic system (the lemmas, corollaries, and theorems). They statements which "make sense." An axiomatic system is complete if a truth value can be put on every WFF.

2. Consider $g(t) = 2 + \cos t$. What is the average rate of change of g on the interval $[-\pi, \pi]$.

p 63 # 4b

Well, average rate of change is

$$\frac{g(t_2) - g(t_1)}{t_2 - t_1} = \frac{(2 + \cos(\pi)) - (2 + \cos(-\pi))}{\pi - (-\pi)}$$

$$= \frac{\cos(\pi) - \cos(-\pi)}{2\pi} = \frac{(-1) - (-1)}{2\pi} = 0$$

$\boxed{0}$

3. Evaluate $\lim_{x\to 1} \dfrac{x-1}{\sqrt{x+3}-2}$.

$\boxed{\text{p 74 \# 37}}$ Well,

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} = \lim_{x\to 1} \left(\frac{x-1}{\sqrt{x+3}-2}\right)\left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right)$$

$$= \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4} = \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x-1)}$$

$$= \lim_{x\to 1} \left(\sqrt{x+3}+2\right) = \sqrt{(1)+3}+2 = \sqrt{4}+2 = 4$$

$$\boxed{4}$$

4. State the **formal** definition of limit: $\lim_{x\to x_0} f(x) = L$.

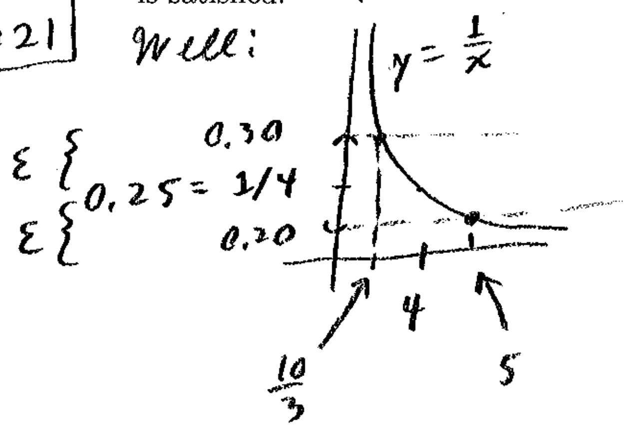
$\boxed{\text{p.77}}$ Let $f(x)$ be defined on an open interval about x_0, except possibly at x_0 itself. We say $\lim_{x\to x_0} f(x) = L$ if, for every $\varepsilon > 0$, there exists a corresponding $\delta > 0$ such that for all x

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$

5. Consider $f(x) = 1/x$. Then $\lim_{x\to 4} f(x) = 1/4$ and in the notation of the definition of limit we have $L = 1/4$ and $x_0 = 4$. For $\epsilon = 0.05$, find a corresponding $\delta > 0$ such that the definition of limit is satisfied.

$\boxed{\text{p 83 \# 21}}$

Well:



We have $\frac{1}{x} = 0.30 \Rightarrow x = \frac{10}{3}$

and $\frac{1}{x} = 0.20 \Rightarrow x = 5$.

So, δ is the minimum of $4 - \frac{10}{3} = \frac{2}{3}$ and $5 - 4 = 1$

$$\boxed{\delta = \frac{2}{3}}$$

6. Consider

$$f(x) = \begin{cases} \sqrt{1-x^2}, & x \in [0,1) \\ 1, & x \in [1,2) \\ 2, & x = 2 \end{cases}.$$

p 91 #9

WARNING!

What is $\lim_{x\to 1} f(x)$? Explain your answer, show details, and heed warning signs.

Well, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{1-x^2} = \lim_{x\to 1^-} \sqrt{\underbrace{(1-x)}_{+}\underbrace{(1+x)}_{+}} = \sqrt{0} = 0$ HEEDED

and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (1) = 1$.

So, $\lim_{x\to 1} f(x)$ does not exist (Theorem 6).

$\boxed{\text{Does not exist}}$

7. (a) State the Intermediate Value Theorem.

p.99

If f is a continuous function on a closed interval $[a,b]$ and if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some $c \in [a,b]$.

(b) If $f(x) = x^3 - 8x + 10$, show that there is a value c for which $f(c) = -\sqrt{3}$.

p102 #57b

Well, f is a polynomial function and so is continuous everywhere. $f(-4) = (-4)^3 - 8(-4) + 10$
$= -64 + 32 + 10 = -22 < -\sqrt{3}$

and $f(0) = (0)^3 - 8(0) + 10 = 10 > -\sqrt{3}$. So by the Intermediate Value Theorem, there exists $c \in [-4, 0]$ such that $f(c) = -\sqrt{3}$.

8. Find the horizontal and vertical asymptotes of $f(x) = \dfrac{x^2-1}{x^2+x-2}$. Graph $y = f(x)$ in a way that reflects the asymptotes of the function.

H.A.: $\displaystyle\lim_{x\to\infty}\left(\frac{x^2-1}{x^2+x-2}\right) = \lim_{x\to\infty}\left(\frac{x^2-1}{x^2+x-2}\right)\left(\frac{1/x^2}{1/x^2}\right) = \frac{\left(\lim_{x\to\infty} 1\right) - \left(\lim_{x\to\infty}\frac{1}{x}\right)^2}{\left(\lim_{x\to\infty} 1\right) + \left(\lim_{x\to\infty}\frac{1}{x}\right)^2 - 2\left(\lim_{x\to\infty}\frac{1}{x}\right)^2}$

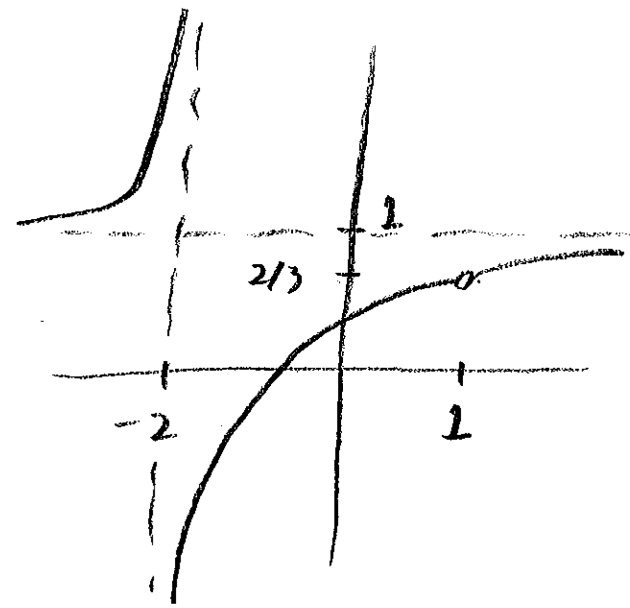
$\displaystyle = \frac{1-0}{1+0-2(0)} = 1.$ $\boxed{\text{So } y = 1 \text{ is H.A.}}$

V.A. $\displaystyle f(x) = \frac{(x-1)(x+1)}{(x+2)(x-1)} = \frac{x+1}{x+2}$ if $x \neq 1$.

$\displaystyle\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-}\left(\frac{x+1}{x+2}\right) \frac{(-)}{(-)} \; +\infty$

$\displaystyle\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+}\left(\frac{x+1}{x+2}\right) \frac{(-)}{(+)} \; -\infty$

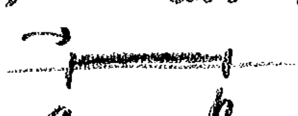
$\displaystyle\lim_{x\to 1} f(x) = \lim_{x\to 1}\left(\frac{x+1}{x+2}\right) = \frac{2}{3}.$



Bonus 1. Is it true that if you stretch a rubber band by moving one end to the right and the other to the left, some point of the band will end up in its original position? Give reasons for your answer.

p103 #66

Let the rubber band lie on the interval $[a, b]$. Define $f(x)$ on $[a,b]$ so that point x is moved to point $f(x)$ under the stretching. So f is continuous on $[a,b]$. Consider $g(x) = f(x) - x$. Then g is cont. on $[a,b]$ and if $g(c) = 0$ then $f(c) = c$ and point c is in its original position. We know $g(a) = f(a) - a < 0$ and $g(b) = f(b) - b > 0$. So by the I.V.T. there is a point $c \in [a,b]$ such that $g(c) = 0$ and so point c is unmoved.

RUBBER BAND 

STRETCHED