

1.1 #21

Find the domain of $y = \frac{x+3}{4-\sqrt{x^2-9}}$.

Solution

Let $y = f(x)$. We can't divide by 0 or take square roots of negatives.

We have division by 0 when

$4 - \sqrt{x^2 - 9} = 0$. So we solve this:

$$\sqrt{x^2 - 9} = 4 \text{ or } (\sqrt{x^2 - 9})^2 = 4^2$$

$$\text{or } x^2 - 9 = 16 \text{ or } x^2 = 25 \text{ or } \sqrt{x^2} = \sqrt{25}$$

$$\text{or } |x| = 5 \text{ or } x = \pm 5.$$

We have square roots of negatives when $x^2 - 9 < 0$. So we solve this:

$x^2 < 9$ or $\sqrt{x^2} < \sqrt{9}$ (because the square root function is an INCREASING function) or $|x| < 3$. This means

$$-3 < x < 3 \text{ (see Appendix A.1) or } x \in (-3, 3).$$

So, the BAD values are -5 , $(-3, 3)$, and 5 . So the domain is everything else:

$$\boxed{(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)}.$$

□