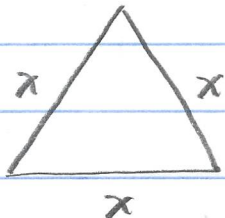


1.1 #9

Express the area and perimeter of an equilateral triangle as a function of the triangle's side length x .

Solution

Well, the triangle is of the form

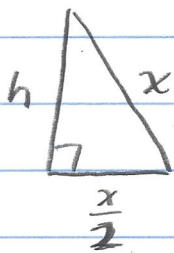


Let the perimeter be P so that

$$\boxed{P(x) = x + x + x = 3x}.$$

Let the area be A . Recall that the area of a triangle is $\frac{1}{2}(\text{BASE})(\text{HEIGHT})$.

Consider



Then by the Pythagorean Theorem we have

$$h^2 + \left(\frac{x}{2}\right)^2 = x^2$$

$$\text{or } h^2 = x^2 - \frac{x^2}{4} = \frac{3x^2}{4}$$

$$\text{or } \sqrt{h^2} = \sqrt{\frac{3x^2}{4}} \text{ or } |h| = \frac{\sqrt{3}|x|}{2}$$

But h and x are distance (so $h \geq 0$ and $x \geq 0$)

and hence $h = \frac{\sqrt{3}}{2}x$. Therefore the

area is $\frac{1}{2}(\text{BASE})(\text{HEIGHT}) = \frac{1}{2}x \frac{\sqrt{3}}{2}x$

$$\text{or } \boxed{A(x) = \frac{\sqrt{3}}{4}x^2}$$