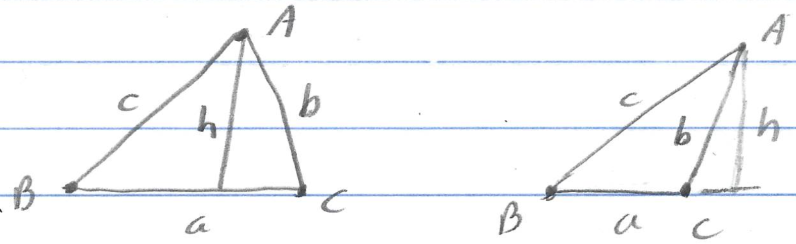
1.3 # 61 Law of Sines The Law of Sines says that if a, b, and c are the sides opposite the angles A, B, and C in a triangle then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

Use the accompanying figures and the identity $\sin(\pi - \theta) = \sin\theta$, if required, to derive the law.

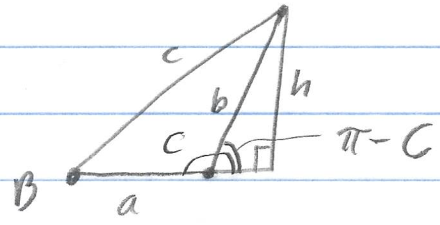


Solution

The figure on the left deals with an acute triangle and on the right with an obtuse triangle. We consider them separately.

On the left, we have (using the right triangles inside the acute triangle) $\sin B = h/c$ and $\sin C = h/b$. So $h = c \sin B$ and $h = b \sin C$. Hence $c \sin B = b \sin C$ or $\frac{\sin B}{b} = \frac{\sin C}{c}$. We can similarly show that these equal $\frac{\sin A}{a}$ (or observe that B and C are arbitrary angles of the triangle so that the result holds for any two angles in the triangle).

On the right (using the right triangles introduced, both with hypotenuse c) we have $\sin B = h/c$ and $\sin(\pi - C) = h/b$:



So $h = c \sin B$ and $h = b \sin(\pi - C) = b \sin C$ (by the given identity). Hence
$c \sin B = b \sin C$ or $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

We can similarly show that these equal $\dfrac{\sin A}{a}$ (or observe that B is an arbitrary acute angle in the triangle so that the result holds for <u>any</u> acute angle in the obtuse triangle). $\square$