

1.5 #21

Find domain and range of

$$f(x) = \frac{1}{2 + e^x}$$

Solution

Notice that  $e^x$  is defined for all  $x \in \mathbb{R}$  (i.e., the domain of  $e^x$  is  $\mathbb{R}$ ).

We could have a problem if the denominator  $2 + e^x$  is 0. BUT

$e^x > 0$  for all  $x \in \mathbb{R}$  (consider the graph of  $y = e^x$ ; see Figure 1.54).

So in  $f$  the variable  $x$  can be

any real number so that the domain of  $f$  is  $\mathbb{R}$ .

Now for the range... since  $e^x > 0$  then the denominator of  $f$  is at least 2:  $2 + e^x > 2$ . Taking reciprocals reverses inequalities (see "order property 6" in

Appendix A1)

so  $\frac{1}{2 + e^x} < \frac{1}{2}$ . Also, since

$e^x > 0$  and  $2 + e^x > 0$  then  $\frac{1}{2 + e^x} > 0$ .

That is  $0 < \frac{1}{2 + e^x} < \frac{1}{2}$ .

So the range is  $(0, 1/2)$ .  $\square$