

1.6 #53

Solve for y : $\ln(y-1) - \ln(2) = x + \ln(x)$.

Solution

We have

$$\ln(y-1) - \ln 2 = \ln\left(\frac{y-1}{2}\right)$$

by Theorem 1.6.1 (2).

$$\ln(y-1) - \ln(2) = x + \ln(x)$$

$$\ln\left(\frac{y-1}{2}\right) = x + \ln(x).$$

$$\text{Hence } e^{\ln\left(\frac{y-1}{2}\right)} = e^{x + \ln(x)}$$

$$\frac{y-1}{2} = e^{x + \ln(x)} \quad (\text{since exp and ln are inverse functions})$$

$$y-1 = 2e^{x + \ln x}$$

$$y = 2e^{x + \ln x} + 1$$

$$= 2e^x e^{\ln x} + 1 \quad \text{by Theorem 1.5.4(1)}$$

$$= 2e^x(x) + 1 \quad \text{since exp and ln are inverse functions}$$

$$\text{or } \boxed{y = 2xe^x + 1.} \quad \square$$