1.6 #85 <u>Radioactive Decay</u>
The half-life of a certain radioactive substance is 12 hours. There are 8 grams present initially. (a) Express the amount of substance remaining as a function of time t. (b) When will there be 1 gram remaining?

<u>Solution</u>

We have $y = y_0 e^{kt}$ for radioactive decay where y_0 is the initial amount present.

(a) We are told the half-life is 12 hours, so $y = y_0/2$ when $t = 12$. We use this to find k: $y = y_0/2 = y_0 e^{k(12)}$

or $\frac{1}{2} = e^{12k}$ or $\ln\left(\frac{1}{2}\right) = \ln\left(e^{12k}\right)$

or $\ln(2^{-1}) = 12k$ or $k = -\ln(2)/12$.

Since the initial amount is $y_0 = 8$ grams then the amount remaining as a function of t is $\boxed{y = 8e^{(-\ln(2)/12)t} \text{ grams}}$.

(b) We have $y = 1$ when $1 = 8e^{(-\ln(2)/12)t}$

or $\frac{1}{8} = e^{(-\ln(2)/12)t}$ or $\ln\left(\frac{1}{8}\right) = \ln\left(e^{(-\ln(2)/12)t}\right)$

or $-\ln(8) = \frac{-\ln(2)}{12} t$ or $t = \frac{12\ln(8)}{\ln(2)} = \frac{12\ln(2^3)}{\ln(2)}$

or $t = \frac{36\ln(2)}{\ln(2)} = \boxed{36 \text{ hours}}$. □