2.1.5 Find the average rate of change of $R(\theta) = \sqrt{4\theta+1}$ for $\theta \in [0,2]$.

Solution

By definition, the average rate of change of $y = R(\theta)$ with respect to θ over the interval $[\theta_1, \theta_2]$ is

$$\frac{\Delta R}{\Delta \theta} = \frac{R(\theta_2) - R(\theta_1)}{\theta_2 - \theta_1} = \frac{R(\theta_1 + h) - R(\theta_1)}{h}$$

where $h = \theta_2 - \theta_1$.

Here, $R(\theta) = \sqrt{4\theta+1}$, $\theta_1 = 0$, $\theta_2 = 2$, and $h = \theta_2 - \theta_1 = 2 - 0 = 2$. So the average rate of change of R on $[0,2]$ is

$$\frac{\Delta R}{\Delta \theta} = \frac{R(\theta_2) - R(\theta_1)}{\theta_2 - \theta_1} = \frac{R(2) - R(0)}{2 - 0}$$

$$= \frac{\sqrt{4(2)+1} - \sqrt{4(0)+1}}{2-0} = \frac{\sqrt{9} - \sqrt{1}}{2} = \frac{3-1}{2} = \boxed{1}.$$