2.2.37 Evaluate $\displaystyle\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2}$.

Solution

We multiply by the CONJUGATE of the denominator (divided by itself):

$$\lim_{x \to 1} \frac{(x-1)}{\sqrt{x+3}-2}$$

$$= \lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2}\left(\frac{-\sqrt{x+3}-2}{-\sqrt{x+3}-2}\right)$$ multiplying by 1

$$= \lim_{x \to 1} \left(\frac{(x-1)}{\sqrt{x+3}-2}\right)\left(\frac{-(\sqrt{x+3}+2)}{-(\sqrt{x+3}+2)}\right)$$

$$= \lim_{x \to 2} \frac{(x-1)(\sqrt{x+3}+2)}{(\sqrt{x+3})^2-(2)^2} = \lim_{x \to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4}$$

$$= \lim_{x \to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x-1)}$$ Factoring

$$= \lim_{x \to 1} \left(\sqrt{x+3}+2\right)$$ Canceling, justified by Theorem 2.2.A, Dr. Bob's Limit Theorem

$$= \lim_{x \to 2} \sqrt{x+3} + \lim_{x \to 1} (2)$$ by Theorem 2.2.1, Sum Rule

By Theorem 2.2 Substitute $\rightarrow$ $$= \sqrt{\lim_{x \to 1} (x+3)} + 2$$ by Theorem 2.2.1, Root Rule

$$= \sqrt{(1)+3}+2 = \sqrt{4}+2 = \boxed{4} \quad \square$$