2.3.13 Find $\delta > 0$ such that the definition of limit is satisfied for the given ε. [SEE the graph for Exercise 2.3.13.]

Solution

We have $L = 2$, $c = -1$, $\varepsilon = 0.5$, and $f(x) = \frac{2}{\sqrt{-x}}$. We want

$|f(x) - L| < \varepsilon$ (or $\left|\frac{2}{\sqrt{-x}} - 2\right| < 0.5$) ?

whenever $0 < |x - c| < \delta$ (or $0 < |x - (-1)| < \delta$).

For $\left|\frac{2}{\sqrt{-x}} - 2\right| < 0.5$ we have

$$-0.5 < \frac{2}{\sqrt{-x}} - 2 < 0.5 \text{ or } 1.5 < \frac{2}{\sqrt{-x}} < 2.5,$$

or $1.5 < f(x) < 2.5$. From the graph, we need $\frac{-16}{9} < x < \frac{-16}{25}$. BUT, to find δ we take the smaller of the two distances $\frac{-16}{25} - (-1) = \frac{-16+25}{25} = \frac{9}{25}$

and $(-1) - \left(\frac{-16}{9}\right) = \frac{-9+16}{9} = \frac{7}{9}$.

Hence, we take $\boxed{\delta = 7/25}$. □