2.3.19 Consider $f(x) = \sqrt{19-x}$, $L = 3$, $c = 10$, and $\varepsilon = 1$. Find an open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then give a value for $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

<u>Solution</u>

First, notice that the domain of $f(x) = \sqrt{19-x}$ is $x \leq 19$. The inequality $|f(x) - L| < \varepsilon$ or $|\sqrt{19-x} - 3| < 1$ is equivalent to $-1 < \sqrt{19-x} - 3 < 1$ or $2 \leq \sqrt{19-x} < 4$ or (since the squaring function is an increasing function for positive inputs) $2^2 < (\sqrt{19-x})^2 < 4^2$ or $4 < 19 - x < 16$ or $-15 < -x < -3$ or $15 > x > 3$. So the desired inequality holds for $\boxed{x \in (3, 15)}$.

Now the distance from $c = 10$ to 3 is $\delta_1 = 7$ and the distance from $c = 10$ to 15 is $\delta_2 = 5$. So we choose δ as the smaller of δ_1 and δ_2; that is, we take $\boxed{\delta = 5.}$ Then for $0 < |x - c| = |x - 10| < \delta = 5$, we have $x \in (5, 15) \subset (3, 15)$ and so the inequality holds as desired. $\square$