2.3.37 Prove $\lim_{x \to 4} (9-x) = 5$.

<u>Proof</u>

We have $f(x) = 9-x$, $c=4$, and $L=5$. Notice that f is defined on an open interval containing $c=4$ (say the interval $(-\infty, \infty)$). Let $\varepsilon > 0$.
[THINKING (not part of proof): We want $|f(x) - L| < \varepsilon$ or $|(9-x)-5| < \varepsilon$ or $|4-x| = |x-4| < \varepsilon$. With $\delta = \varepsilon > 0$ we have $0 < |x-c| < \delta$ or $0 < |x-4| < \delta$ implies $0 < |x-4| < \delta = \varepsilon$ and then we can reverse the above computations.]
Let $\delta = \varepsilon$. If $0 < |x-c| < \delta$ then $0 < |x-4| < \delta$ or $0 < |x-4| < \varepsilon$, which implies $|x-4| < \varepsilon$. Now $|x-4| < \varepsilon$ is equivalent to $|4-x| < \varepsilon$ or $|(9-x)-5| < \varepsilon$ or $|f(x) - L| < \varepsilon$.
Therefore, by the definition of limit, $\lim_{x \to c} f(x) = L$ or $\lim_{x \to 4} (9-x) = 5$, as claimed. ■