2.4.17 (a) Evaluate $\lim_{x \to -2^+} (x+3)\frac{|x+2|}{x+2}$. Explain.

(b) Evaluate $\lim_{x \to -2^-} (x+3)\frac{|x+2|}{x+2}$. Explain.

<u>Solution</u>

(a) For $x \to -2^+$ we have that x is close to -2 and greater than -2. So $x+2>0$ and hence $|x+2| = x+2$. So

$$\lim_{x \to -2^+} (x+3)\frac{|x+2|}{x+2} = \lim_{x \to -2^+} (x+3)\frac{x+2}{x+2}$$

since $|x+2| = x+2$ here

$$= \lim_{x \to -2^+} (x+3)$$ by Dr. Bob's Limit Theorem (Thm 2.2.A) applied to one-sided limits

$$= (-2)+3$$ by Theorem 2.2, "Limits of Polynomials" applied to one-sided limits

$$= \boxed{1}.$$ □

(b) For $x \to -2^-$ we have that x is close to -2 and less than -2. So $x+2<0$ and hence $|x+2| = -(x+2)$. So

$$\lim_{x \to -2^-} (x+3)\frac{|x+2|}{x+2} = \lim_{x \to -2^-} (x+3)\frac{-(x+2)}{x+2}$$

since $|x+2| = -(x+2)$ here

$$= \lim_{x \to -2^-} (x+3)(-1)$$ by Dr. Bob's Limit Theorem (Theorem 2.2.A) applied to one-sided limits

$= ((-2)+3)(-1)$ by Theorem 2.2, "Limits of Polynomials" applied to one-sided limits

$= \boxed{-1}$. $\square$

<u>NOTE</u> Since $\lim_{x \to -2^-} (x+3)\frac{|x+2|}{x+2} \neq \lim_{x \to -2^+} (x+3)\frac{|x+2|}{x+2}$

then by Theorem 4.6, "Relation Between One-Sided and Two-Sided Limits", we have that

$\lim_{x \to -2} (x+3)\frac{|x+2|}{x+2}$ does not exist.